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★**Twelve landmarks of twentieth-century analysis.**
Illustrated by Michaël Monerau.
Translated from the 2009 French original by Danièle Gibbons and Greg Gibbons.
With a foreword by Gilles Godefroy.
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This wide-ranging and engaging expository monograph began as a series of topics seminars the authors, Denis Choimet and Hervé Queffélec, ran at the Lycée Clemenceau in Nantes starting a decade and a half ago. Developed into a book—first in French [*Analyse mathématique: grands théorèmes du vingtième siècle*, Calvage amp; Mounet, Montrouge, 2009] and now in English translation—it makes a distinctive contribution to the advanced pedagogical literature in analysis, offering a bridge to advanced topics for suitably prepared students and a fresh perspective for professors of analysis on their field.

In thirteen chapters, Choimet and Queffélec rove across a rich and interconnected selection of topics from twentieth-century analysis. The book opens with a litany of Littlewood, detailing a series of Tauberian theorems (on convergence conditions for power series in a single variable) beginning with a structured reading of Littlewood's 1911 paper that defined a far-reaching program of research on the subject. Across the first three chapters, the authors exhibit a broad range of methods and implications from this Tauberian program, with special attention given to Fourier analysis and the Prime Number Theorem. The next suite of chapters examine a variety of topics in the theory of functions and measures, showing the authors' pedagogical flair in drawing out the significance of theorems related to discontinuities and related properties of derivatives (chapter 4), probability and existence theorems (chapter 5), and measure-theoretic paradoxes (chapter 6). Choimet and Queffélec then dig into certain special functions: Riemann's 'other' function ($\sum_{n=1}^{\infty}\frac{\sin(n^2\pi x)}{n^2}$) in chapter 7, the partition function in chapter 8, and the Jacobi θ_0 function in chapter 9. Chapter 10 focuses on a conjecture that grew from one of Littlewood's last collaborations with Hardy, on the norms of exponential sums. The book then closes (without an explicit denouement) in three chapters on Banach algebras: a preparatory discussion of Gelfand's formulation of the subject (chapter 11), followed by a lengthy exposition of Carleson's corona theorem and its prerequisites (chapter 12), and Lindenstrauss's study of complementation in Banach spaces (chapter 13).

The text's origin as both extracurricular enrichment and an invitation to the pleasures and difficulties of modern analysis thoroughly inflects the authors' style, methods, and subject matter. Despite occasional nods to novice readers, the authors take a good deal of preparation for granted, elaborating definitions and basic principles where it suits their explanations rather than where a strictly grounded development might dictate. The authors furnish each chapter with a set of exercises, accompanied by hints but not full solutions in the endmatter, that guide the reader to develop further consequences of that chapter's material. Throughout, they deploy a sturdy array of primary sources and reference literature, with a judicious assortment of references for further reading. An index of notations at the end and frequent cross-references make it possible to skip around the text as interest dictates. The authors excel at explaining the motivations and reasoning behind a great variety of approaches in analysis, provided one has the requisite familiarity with the graduate analysis curriculum to follow along. Among their many expository strengths, I appreciated Choimet and Queffélec's effective use of argument by wishful thinking, which helps them build up useful intuitions before resolving the problem at hand. The work is historical in the sense of curating and synthesizing past works with a certain attention to what was known and prioritized by mathematicians in the past. But the discussion is squarely mathematical, and its interest lies in its development of mathematical themes rather than in its piecemeal depiction of the history of analysis in the twentieth century.

The translators Danièle and Greg Gibbons have produced a careful translation that retains just enough of a French accent in word choice and phrasing for authenticity while conveying what is, by the standards of this genre, a refreshingly lively and playful authorial voice. The text's sprinkling of quirks and inconsistencies are not distracting, though one sometimes wishes for more restraint in the use of footnotes. Michaël Monerau's illustrations are simple and effective, while Gilles Godefroy's new foreword for this edition is brief and forgettable.

My primary complaint is not specific to this work. By writing yet another contribution (quite an effective contribution, to be sure) to the teeming expository literature on 'great theorems by great men', the authors inevitably contribute to a certain narrowness in mathematicians' collective memory. When we choose our landmarks from the past, we choose at the same time how to see our discipline in the present and for the future. One gets a vivid sense in this book of the intellectual vibrancy of twentieth-century mathematics, but less of the personal vibrancy that put analysis on the leading edge of mathematicians' markedly diversifying and globalizing profile in that same century. Even within this volume's particular topical focus, for instance, Mary Cartwright might count as a conspicuous omission. I hope Choimet and Queffélec's perceptive interpretations of Littlewood, Hardy, Wiener, Banach, Gelfand, Carleson, and other celebrated figures might inspire equally perceptive appreciations of those still without a firm place in the analytic canon who may bear more complete witness to what analysis has been and can be.

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