

# Chapter 4

## Translating Euclid's diagrams into English, 1551–1571

Michael J. Barany

**Abstract** The years 1551–1571 saw the first published translations of Euclid's *Elements* in the English language. Euclid's first English translators had to translate not just his words, but his entire system of geometry for a vernacular public unversed in a method and study hitherto 'locked up in strange tongues.' Throughout its written and printed history, diagrams have been crucial features of Euclid's text. This paper considers the variety of diagrammatic approaches used in these first English translations, arguing that the strategic inclusion and exclusion of points, lines, letters, and labels, along with depictions of surveying instruments and landscapes, played crucial roles in establishing the authors' voices, vocabularies, methodologies, and mathematical philosophies. Using simple but polysemic objects such as points and lines, and appealing to familiar practices such as drawing, using a compass, and surveying a field, Euclid's translators projected and enforced an image of a geometry which could be seen to be already present and meaningful. Their diagrams, rather than being *mere* illustrations, played indispensable roles in establishing the new English geometry.

**Key words:** Euclid's *Elements*; Robert Recorde, John Dee, Henry Billingsley, Leonard Digges, Thomas Digges, Geometry, Translation, Diagrams, Representation

### 4.1 Translating Diagrams

In 1551, Robert Recorde published England's first surviving vernacular textbook on the principles of Euclidean geometry, *Pathway to Knowledge*.<sup>1</sup> Recorde's

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Princeton University Program in History of Science, mbarany@princeton.edu

<sup>1</sup> Taylor (1954, 14–15, 312) discusses earlier surveying texts and a possible prior translation

text presents the definitions, axioms, postulates, and propositions of the first four books of Euclid's *Elements* in such a way that "the simple reader might not justly complain of hardnes or obscuritee."<sup>2</sup> Making Euclid's ancient geometry newly accessible to his vernacular readers, Recorde's translation involved more than mere linguistic substitution and coinage. For Recorde was not merely translating between languages, but across concepts, idioms, places, times, social positions, and professions. Euclid's new tongue grew out of a translation in the fullest sense of the word.

Henry Billingsley completed his own edition of the *Elements* in 1570. The volume has been identified by Heath (1956) as "the first and most important translation" of Euclid's *Elements* into English (109). Billingsley's text incorporated "Scholies, Annotations, and Inventions, of the best Mathematiciens, both of time past, and in this our age" and a "very fruitfull Præface" by his collaborator John Dee.<sup>3</sup> The text was produced for, as Dee writes, "unlatined people, and not Universitie Scholers," and included all fifteen books then attributed to Euclid of Megara.<sup>4</sup> The following year, Thomas Digges posthumously published his father Leonard's tripartite geometrical practice, *Pantometria*, appending a preface and his own discourse on geometrical solids. *Pantometria* emphasizes surveying and the military arts, while Thomas's contribution concerns "matters only new, rare and difficile."<sup>5</sup>

This paper focuses on one aspect of these five authors' translations: their diagrams. Geometric diagrams and figures had been present even in the first print editions of the *Elements*, dating to the printer Erhard Ratdolt's 1482 volume, and the inclusion of illustrative diagrams was a standard feature for geometric texts.<sup>6</sup> Indeed, Euclid's *Elements* can scarcely be understood without the aid of geometric illustrations, and the visual vocabulary of the *Elements* had long been established as a central feature of geometric learning. Even so, the sheer variety of diagrammatic approaches used in the first English vernacular translations of the *Elements* indicates that the choice of how to illustrate one's text was no trivial matter.

I use the word 'diagrams' here in an unusual and anachronistic sense, but one which seems to me the most justifiable for the discussion that will fol-

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of Euclid.

<sup>2</sup> Sig. *a1r*. Page citations are according to Gaskell (1972). I use Johnson and Larkey's (1935) convention of preserving spellings while sometimes modernizing typography by, for example, expanding contractions and converting 'u's to 'v's where appropriate. Emphasis in quotations is the quoted author's.

<sup>3</sup> Sig. [fist]1<sup>r</sup>.

<sup>4</sup> Sig. *A3v*. The *Elements* are now typically attributed to Euclid of Alexandria, instead of Megara, and only the first thirteen of the books included in Billingsley's translation are considered to be of Euclid's authorship (Heath 1956, 3–5).

<sup>5</sup> Sig. *S4v*.

<sup>6</sup> On manuscript geometric diagrams, see Keller (2005) and De Young (2005, 2009).

low. The diagrams considered here comprise all manner of visual para-text – including illustrations of definitions, constructions, proof figures, and drawings of instruments – designed to facilitate geometric understanding in these translations of the *Elements*. Such an expansive view is necessary for two principal reasons. The first is that by construing diagrams broadly one is better able to account for the diversity of illustrative approaches used by the different authors. Drawing from the same representational traditions found in past editions of the *Elements*, each translator took a different approach to rendering the visual and geometric meaning in those texts for his vernacular readers.

More importantly, a detailed reading of the visual vocabularies in these starkly varied texts cannot help but undermine any narrow circumscription of what counts as a diagram. One finds in our texts a variety of meanings for terms such as ‘figure’ and ‘example’ as well as a variety of uses for illustrations of different sorts. As all of these works were produced in print, the safest delineation seems to be that between conventional alphabetical text and other printed illustrations, including their captions. There, the extra work of producing figures and arranging the rest of the text around them suggests a special place for such images in our consideration of these translations. In these figures, we shall see the junctures where our authors found, for a diversity of reasons, that words did not suffice.<sup>7</sup>

Diagrams, for our authors, were integral means of establishing a new English geometry which was simultaneously comprehensible, even familiar, to its vernacular readers and a part of an ancient mathematical tradition. The next section provides some further necessary context for the authors and texts under consideration, exploring what it means to translate the *Elements*. I then consider Recorde and Billingsley's uses of diagrams, first in turn and then in comparison, and contrast these uses to those of Dee and Leonard and Thomas Digges. Finally, I synthesize these observations by comparing how each author establishes the definitions for parallel lines and the simple geometric point. In these texts, I argue, the strategic inclusion and exclusion of points, lines, letters, and labels, along with the depiction of instruments and landscapes, figured crucially in the establishment of the authors' voices, vocabularies, methodologies, and mathematical philosophies.

## 4.2 Translating Euclid

Between the first geometrical writings of Recorde, Billingsley, Dee, and Leonard and Thomas Digges, one finds the foundations of English vernac-

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<sup>7</sup> This is not to discount the crucial role of printers and engravers in preparing diagrams. The texts engage sufficiently with their illustrations that it is safe to presume substantial involvement on the part of the authors, but there are also signs (particularly in errors or omissions) that suggest the limits of such involvement. For the remainder of this paper, I

ular geometry. Billingsley was a wealthy merchant and translator of several genres. His collaboration with Dee connected him to a closely interlinked Tudor tetrumvirate of English mathematics. Leonard and Thomas Digges both use elements of Recorde's terminology in their work, and Dee had worked directly with Recorde's arithmetical text *The Ground of Artes*.<sup>8</sup> Dee and the elder Digges knew each other personally, and the younger Digges was a pupil of Dee, who became Thomas's "second mathematical father" after Leonard's death.<sup>9</sup>

While Billingsley's is the only work of these five authors typically counted among translations of Euclid's *Elements*, and is certainly the most complete and literal of the group, the texts of each played pivotal roles in shaping Euclid's reception in England. *Pathway* offered many new geometric terms for geometry's new language, and, following Proclus, was the first modern text to classify Euclid's propositions as either constructions or theorems.<sup>10</sup> Dee's preface presented a taxonomy of the mathematical sciences and was among the most influential mathematical texts of the late sixteenth century.<sup>11</sup> *Pantometria* offered a definitive bridge between practical and theoretical geometry from an author already widely read by practical users of the art.<sup>12</sup> Beginning with "Elementes of Geometrie, or Diffinitions," its Euclidean allusions and aspirations permeate the text.<sup>13</sup> Thomas Digges's treatise on geometric solids was the first of many works securing his place as one of England's most eminent mathematicians.<sup>14</sup> All five authors incorporate Euclid's style and content, implicitly or explicitly, into their own.

In this respect, all five should be counted among the first English translators of Euclid's *Elements*.<sup>15</sup> One must remember that before the work of these translators there was no geometry, as such, outside the universities in England.<sup>16</sup> Our authors realized the novelty and significance of what they were creating.<sup>17</sup> "For nother is there anie matter more straunge in the englishe tungue," Recorde explains, "then this where of never booke was written be-

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will attribute the constellation of authorships underlying the diagrams to the works' official authors.

<sup>8</sup> Johnson (1944, 132), Roberts (2004), Johnston (2004a, 2004b), Easton (1967, 515), Heninger (1969, 109).

<sup>9</sup> Digges (1573, Sig.A2<sup>r</sup>), Johnston (2004b), Johnson (1936, 398–399), Taylor (1954, 166).

<sup>10</sup> Johnson and Larkey (1935, 68), Johnson (1944). Johnson identifies 'straight line' as the only one of Recorde's coinages to have survived to the present day.

<sup>11</sup> Roberts (2004, 200), Taylor (1954, 170–171, 320). <sup>12</sup> McRae (1993, 345). <sup>13</sup> Sig.B1<sup>r</sup>.

<sup>14</sup> Johnston (2004b).

<sup>15</sup> Euclid's work was first printed in Latin in 1482 and its first full vernacular rendering was in Italian some sixty years later. Heath (1956, 97, 106). On English translations of the *Elements*, see Barrow-Green (2006, esp. 3–7).

<sup>16</sup> Feingold (1984, 178), for instance, credits these very authors with introducing higher mathematics to "London's practitioners as well as its scholars."

<sup>17</sup> Bennett (1986, 10–11); Hill (1998, 253).

fore now, in that tuncue.”<sup>18</sup> In his 1556 *Tectonicon*, an elementary technical treatise, Leonard Digges explains his intention to write his forthcoming *Pantometria* for “all maner men of this realme” and for making accessible “those rules hidde, and as it were locked up in straunge tongues.”<sup>19</sup> Billingsley desired that “our Englishe tounge shall no lesse be enriched with good Authors, then are other straunge tounge.”<sup>20</sup>

But these authors did not aim merely to reproduce the same obscure knowledge in a different language. Rather, as Thomas Digges writes of his father's intentions, works were “compiled in the Englishe tong, desiring rather with plaine and profitable conclusions to store his native language and benefite his Countrey men, than by publishing in the Latin rare and curiouse demonstrations, to purchase fame among straungers.”<sup>21</sup> In the second edition of *Pantometria*, Digges further declares his resolve to publish “onely in my Native Language: Aswell to make the benefite thereof the more private to my Countrey men, as also to make thereby other Nations to affect as much our Language as my selfe have desired to learne the Highe Dutche.”<sup>22</sup> Recorde and Thomas Digges both saw mathematics and a mathematically literate public as important elements of statecraft.<sup>23</sup> Dee adds the aims that this “*Englishe Geometrie*” would occupy those with sharp wits but lacking philosophical inclinations and simultaneously serve to increase the prestige of university mathematics among the general public.<sup>24</sup>

The geometry our authors made was a local geometry, valid in particular ways for its particular users. It was also, however, a global geometry which, though newly minted, could be traced in the authors' prefaces as far back as Archimedes's defense of Syracuse.<sup>25</sup> Our authors rendered geometric truth for English vernacular audiences by appealing in multiple ways to their audiences' situated and local experiences of the art. Their task was to render as geometry the multifarious knowledges and practices brought to bear by their vernacular readers.

It need hardly be mentioned that the translations did not emerge in a vacuum. Indeed, the annotated text from which Billingsley prepared the bulk of his translation survives to this day – a 1548 edition of Zamberti's Latin translation of Theon's version of the *Elements*.<sup>26</sup> One can infer from the others' writings that they were well read in the mathematics of their contemporaries,

<sup>18</sup> Sig.*a2*<sup>r</sup>. <sup>19</sup> Sig. $\pi$ 2<sup>r</sup>. <sup>20</sup> Sig.[fist]3<sup>r</sup>. <sup>21</sup> Sig.A4<sup>v</sup>.

<sup>22</sup> Digges (1591, 176). Recorde, too, published only in English. Johnson and Larkey (1935, 85).

<sup>23</sup> Easton (1966, 354–355; 1967, 523), Hill (1998, 256), Feingold (1984, 186, 206–207).

<sup>24</sup> Sig.A4<sup>r</sup>. <sup>25</sup> Recorde Sig.†2<sup>v</sup>–†3<sup>v</sup>, Digges Sig.A3<sup>v</sup>, Dee Sig.C4<sup>v</sup>.

<sup>26</sup> Archibald (1950) reviews the history and historiography of Billingsley's sources. See also Feingold (1984, 158).

both in England and on the Continent. Recorde, for instance, refers in his text to claims made by German near-contemporary Albrecht Dürer.<sup>27</sup> Moreover, the personal and scholarly ties joining the translators makes it reasonable to assume that they had access to a similar corpus of mathematical works. Beyond that, however, it is difficult to untangle the variety of sources upon which they drew for their geometric works – certainly, such a task is beyond the scope of the present paper.

Even in the case of their diagrams, which could presumably be transferred more recognizably from their various sources, one can assert little beyond the observation that, broadly speaking, there is little that is particularly innovative about the representational strategies employed in the English translations. Features identified in the discussion below can, with few exceptions, be found in prior works in other languages, both from the overtly Euclidean volumes<sup>28</sup> and from other geometric texts.<sup>29</sup> Each of the English translations combines images and motifs identifiable in multiple prior works. In light of the common visual vocabulary upon which our five translators could draw, along with the convergence in pedagogical intent among all but Thomas Digges, it is all the more remarkable how different their works appear.

My analysis takes these authors' aims and dispositions, drawn largely from the writings of the authors themselves, as its starting point. My goal is not to evaluate the success or failure of these authors, nor to assess their influence and influences, nor even to address the surely complicated matters of authorship and responsibility for the various words and figures of their respective texts. Taking the works' attributed authors at their word, the ensuing analysis explores what can be learned by contrasting the different representations as they stand before us.

Each author set out to fashion a new English geometry on the back of Euclid's *Elements*. I shall interrogate their texts in order to shed light on what they deemed necessary in order to accomplish such a monumental task. The result will not account for the images, nor will it be simply an accounting of them. Rather, it will comprise a first inquiry into how the images account for geometry. My question shall be how Euclid's diagrams, in our broad sense of the word, were translated for an English vernacular readership. How, in other words, was the visual vocabulary of the *Elements* made meaningful for this (at least purportedly) new audience? The different strategies employed by the texts under consideration offer a view of the choices each author made and the range of strategies each author set aside in his attempt to render a vernacular geometry.

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<sup>27</sup> Sig.A4<sup>v</sup>. <sup>28</sup> For example, Ratdolt (1482), Pacioli (1523), Grynaeus (1533), Hirschvogel (1543), Benedetti (1553), or Xylander (1562).

<sup>29</sup> For example, Nemorarius (1533), Fine (1544), Cardano (1554), or Frisius (1557).

### 4.3 Recorde's English Geometry

Robert Recorde's *Pathway to Knowledge* presented a geometry steeped in the familiar trades and practices of men in all walks of life, whether or not they were potential readers of his book. To help establish that geometry truly was everywhere, his preface lists no fewer than sixteen 'unlearned' professions which, he claimed, already relied on the subject.<sup>30</sup> Users of geometry include the commoner, the deity, the contemporary, the ancient, and (implicitly) everyone in between. "Ceres and Pallas," for instance, join a congregation of Ancient figures who "were called goddes" for teaching little more than geometry's applications, and Galen "coulede never cure well a rounde ulcere, till reason geometricall didde teache it him."<sup>31</sup>

Yet Recorde's geometry consisted of a mass of terms, methods, and ways of organizing knowledge which had never before appeared in the English language. To bridge this gap between theoretical knowledge and purported practice, Recorde enlisted both words and images. Diagrams and illustrations in the early pages of Recorde's exposition are laden with extra contextualizing details. Thus, 'A twiste line' is shown wrapped about a column and a right angle in a construction is shown against a drafting square. Even abstract shapes are drawn with hatching in order to give a sense of depth and form (figures 4.1(a)–(c)).

Definitions are illustrated with figures that can also stand alone without the expository text surrounding them. Typographically differentiated terms from Recorde's exposition match copious labels attached to the figured objects being defined.<sup>32</sup> Such typographical cues create an explicit link between text and figure, and in so doing they establish parallel functions for the textual definition and its associated diagram. Thus, the components of the figure are not just semantically but also structurally mapped onto the components of the definition. The structural authority thus acquired by the definitional diagram makes it a credible stand-in for its textual counterpart. Particularly in a setting where geometry's rhetorical formulae had not entered the vernacular, such an elevated role for diagrams offered a crucial conceptual bridge for Recorde's readers.

Nor did Recorde's diagrams stop at merely illustrating individual terms. In many cases, definitional figures show the multiplicity and variety encompassed by the term or terms in question – something the written text would be hard-pressed to do without distractingly wordy descriptions. Individual concepts are instantiated, in Recorde's illustrations, by a sometimes-vast variety of

<sup>30</sup> Sig.[ez]4<sup>v</sup>–†1<sup>v</sup>.   <sup>31</sup> Sig.†1<sup>v</sup>–2<sup>r</sup>.

<sup>32</sup> See figures 4.1(a)(b)(d) and 4.27 (below). In 4.1(d), the label for 'A corde' was included without the cord even being drawn into the figure.

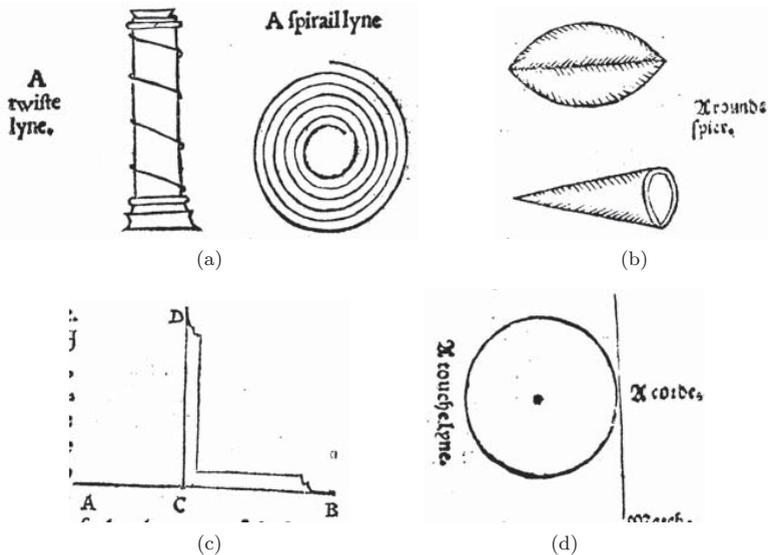


Fig. 4.1: Definition and construction figures with contextualizing details and internal captions from Recorde (1551): (a) twist and spiral lines, Sig.A4<sup>v</sup>; (b) two three-dimensional shapes, Sig.C1<sup>r</sup>; (c) a construction using a drafting square, Sig.D1<sup>v</sup>; (d) a tangent ('touche') line, Sig.B1<sup>r</sup>

cases. In some places, images are reused to illustrate multiple phenomena, as when Recorde's exemplary 'spirail line' joins a dizzying array of 'croked' ones (figure 4.2). Here, expediency for the printer reinforces the mathematical principle that the same object can belong to many geometric classes.

According to Recorde, the diagrams establish abstract geometric concepts on the basis of "such undowbtfull and sensible principles."<sup>33</sup> It is important that this approach is emphasized at an early stage in the text. On the first page of his definitions, for example, Recorde explains that a line is composed of points<sup>34</sup> by saying that "if you with your pen will set in more other prickes betweene everye two of these [in the dotted line above], then wil it be a line."<sup>35</sup> His demonstration purports to explain the composition of both the particular dotted and solid lines on the page and the general concept of a geometric line. He thus transforms the familiar action of drawing points and

<sup>33</sup> Sig.c1<sup>r</sup>.

<sup>34</sup> See Alexander (1995, 580–581) for contemporary disputes over the composition of the continuum.

<sup>35</sup> Sig.A1<sup>r</sup>.

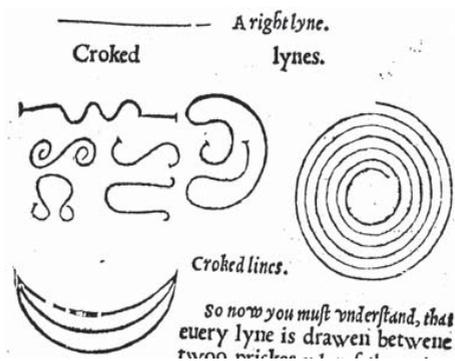


Fig. 4.2: Definition figure for crooked lines from Recorde (1551, Sig.A1<sup>v</sup>)

lines from the mere production of marks on the page into the production of geometry. The reader can see that all lines are composed of points through a practice so intuitive that it need not actually be performed. That such practices need only be performed hypothetically becomes important later on, where assertions which cannot be verified with a few dots of a pen nonetheless inherit the same degree of clarity and obviousness as those which can.

This same reliance on what was readily sensible led Recorde to exercise caution where the diagrams might introduce ambiguities. He explains the dangers of deceptive figures in his discussion of right angles (figure 4.3). Though “angles (as you see) are made partly of streght lines, partly of croked lines, and partly of both together,” his illustrations of right angles show only straight lines, “because it would muche trouble a lerner to judge them: for their true judgment doth appertaine. . . rather to reason then to sense.”<sup>36</sup> Indeed, geometry was, for Recorde, a foundation for reason, not something reason could teach. As the more fundamental subject, geometry was to be grounded in the already-meaningful and already-obvious – that is, it was to be grounded in the purely sensible.

Recorde’s use of an excess of exemplars in order to define geometric entities was and remains a widely employed practice. But Recorde does not stop his parade of multiplicity as he embarks on constructions, which in other contexts are illustrated and discussed in just one putatively generic case. Emphasizing the development of an intuitive comprehension of geometric truths over their rigorous proof-based establishment, Recorde frequently offers multiple constructions for a single proposition or problem. His diagrams show how the construction should be applied in different situations and imply that the ge-

<sup>36</sup> Sig.A1<sup>v</sup>–A2<sup>v</sup>.



Fig. 4.3: Definition figures for right and sharp angles from Recorde (1551, Sig.A2<sup>r</sup>)

ometric conclusion is no mere byproduct of an overly facile case study while suggesting how other geometric case studies might be applied beyond the cases in which Recorde considers them explicitly.

*Pathway* also extends the familiarity-granting depiction of the everyday objects of practical geometry to the constructions later in the text. Thus, Recorde depicts a window arch with a hanging plumbline to accompany three constructions for bisecting a semicircle using different surveying tools, including compasses and drafting squares. He illustrates Euclid's petition to construct a circle from a point and a radius with a picture of a compass, rather than the series of embedded circles, sometimes accompanied by radii, used in nearly all of the other Euclidean texts of his period (figures 4.1(c), 4.4; cf. figure 4.10).

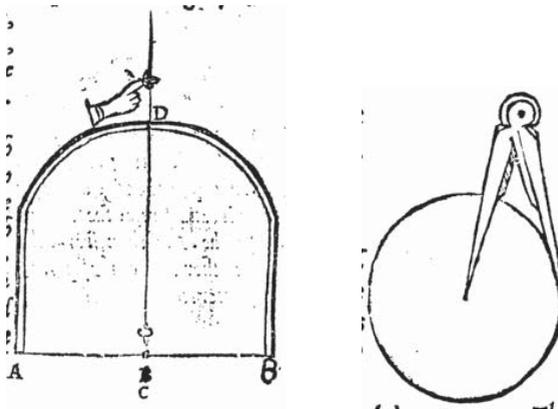


Fig. 4.4: Constructions using, respectively, a plumb line and a compass from Recorde (1551, Sig.D1<sup>r</sup>, b1<sup>r</sup>)

Diagrams can, moreover, indicate information that is omitted in the text, as where Recorde's diagrams show that he intends his theorems about triangles to apply only to ones formed of straight lines, even though his defini-

tions stress that triangles can also be formed using curved, or crooked, lines. Recorde's diagrams rarely include numbers, and while some numbers appear to correspond with their associated measures in the diagram,<sup>37</sup> others correspond hardly at all to the proportions of the drawn figure itself (e.g. figure 4.5). With his proofs, Recorde is explicit about having

drawn in the Linearic examples many times more lines, than be spoken of in the explication of them, whiche is doone to this intent, that if any manne list to learne the demonstrations by harte, . . . those same men should finde the Linearic exaamples to serve for this purpose, and to want no thing needefull to the juste prooffe. . . .<sup>38</sup>

This practice is evident in his figure for the Pythagorean theorem, discussed below (page 141), but we shall also see more examples where pluralistic considerations lead to the inclusion of many more diagram elements than would be used in more conventional Euclidean proofs such as Billingsley's (see figures 4.7 and 4.11).

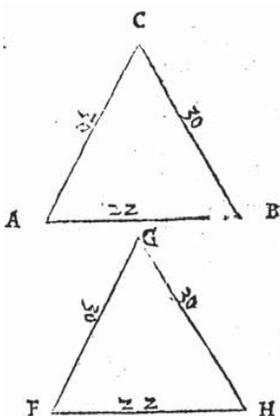


Fig. 4.5: Triangles with specified measures from Recorde (1551, Sig.c1<sup>v</sup>)

Pedagogical through and through, Recorde's text works by guiding the reader through concepts using explicitly exemplary situations. For Euclid's common notions relating to equalities of magnitude, Recorde uses the areas of rectangles and triangles as his case studies (figure 4.6). For the sixth common notion, that two doubles of the same thing are equal to each other, Recorde's diagram includes two copies of the doubled rectangle. These are

<sup>37</sup> e.g. Sig. b2<sup>v</sup> and b3<sup>v</sup>, both of which show tick-marks, and e4<sup>r</sup>, for the Pythagorean theorem, discussed below.

<sup>38</sup> Sig a3<sup>r</sup>-a3<sup>v</sup>.

arranged alongside two larger rectangles formed by joining the smaller ones along different edges, showing both that geometric objects can be doubled in multiple ways and underscoring that either doubling produces the same new area. As with the definitions discussed above, the diagrams for Recorde's common notions aim to establish not just the legitimacy of the claim but also its scope and purpose. Recorde's readers had to be convinced that it was meaningful to compare different ways of doubling an object before examining those comparisons, just as it was necessary to exhibit a multitude of angles and shapes before embarking on their systematic classification.

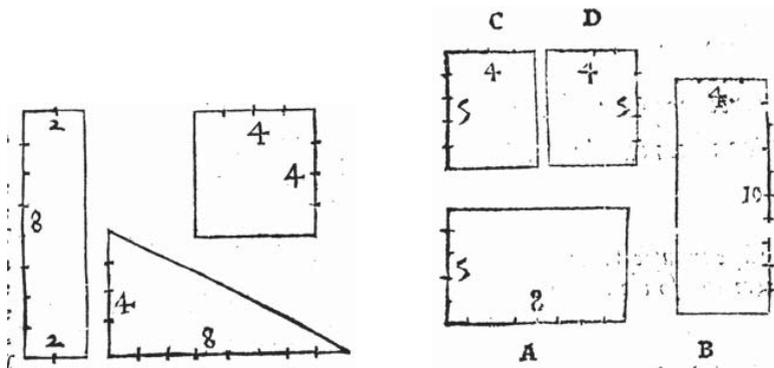


Fig. 4.6: Figures for two common notions from Recorde (1551, Sig.*b2<sup>v</sup>*, *b3<sup>v</sup>*)

Elsewhere, Recorde puts his images to multiple use by illustrating a method of partitioning polygons using parallel lines in a construction involving triangulation (figure 4.7). One diagram indicates how to triangulate simple polygons of increasingly many edges while the other shows a large selection of more complicated polygons which suggests the general applicability of the construction and a practical means of applying it. Finally, Recorde takes care to show why some possible exceptions to his geometric principles are not so. This involves showing variations on a theorem which fail to hold (see figure 4.11 below) and demonstrating how two straight lines cannot enclose a region by showing a regions and non-regions made of different combinations of two curved or straight lines (figure 4.8).

The diagrams in Recorde's *Pathway to Knowledge* are thus made to perform a variety of functions as a crucial supplement to the text. His images establish the legitimacy, meaningfulness, and familiarity of everything from simple geometric objects to relatively complex assertions, constructions, and theorems. Recorde exhibits a geometry addressed to a dazzling array of shapes and objects from both the geometric world and the everyday one. His fig-

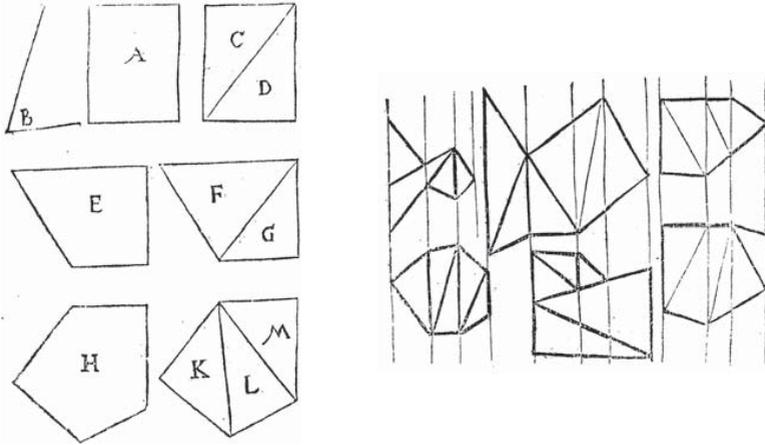


Fig. 4.7: Constructions involving triangulation from Recorde (1551, Sig.E1<sup>v</sup>–E2<sup>r</sup>)

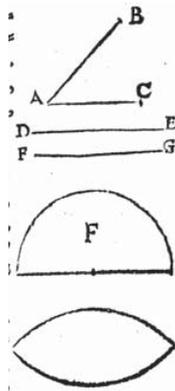


Fig. 4.8: Pairs of lines from Recorde (1551, Sig.b2<sup>r</sup>)

ures justify geometry through its contexts while simultaneously showing how such contexts are to be translated and manipulated according to geometric conventions. Many of these aspects appear in different ways in the figures of subsequent texts, and Recorde's work offers a rich template against which to set later English geometries.

## 4.4 Euclid According to Billingsley

At first glance, the illustrations in Billingsley's compendious edition of the *Elements* are unremarkable. He holds close to what by that time were highly standardized diagrammatic conventions in the manuscript and even print traditions which preceded his contribution. For the purposes of this essay, and without aiming to describe Billingsley's many influences, it should suffice to note that the 'look and feel' of Billingsley's (albeit exceedingly thorough and well-appointed) text does not depart dramatically from other authoritative versions of the *Elements* in circulation at his time.

The text does not arouse our interest for its representational innovations so much as for the means by which it deploys its very unoriginal illustrations to serve an utterly original audience. The work's diagrams and figures must have been the object of much careful consideration. The author attests to the "charge & great travaile" incurred in translating the *Elements*, stating in the text's frontmatter that "I have added easie and plaine declarations and examples by figures, of the definitions."<sup>39</sup> The book is copiously illustrated, and no cost was spared in annexing images to proofs, definitions, scholia, examples, and other textual features. Where proofs span a page-turn, their corresponding diagrams are typically copied over so that they are always visible when following the proof.<sup>40</sup> To help the reader grasp three-dimensional shapes, Billingsley adds to his two-dimensional diagrams a parallel set which use fold-out flaps so that the shapes literally pop out of the page. This latter was perhaps the most distinctive of Billingsley's arsenal of illustrative tools.

Billingsley's representational strategies are best seen in contrast to Recorde's. Although they claim in their prefaces to be writing for similar audiences and to similar ends, it is not hard to see where their purposes diverge. The difference goes all the way down to what sort of geometry they would have their readers learn. As a case in point, contrast Recorde's approach to multiple representations of a triangulation procedure (figure 4.7 above) to Billingsley's treatment of the proof for Euclid's second proposition, which concerns the reproduction of a line segment at a new location.<sup>41</sup> Recorde shows a suitably representative variety of case studies for his procedure, addressing it to increasingly complex polygons in order to inspire confidence in the method's general applicability and point to how it might be so applied. Billingsley, by contrast, exhausts all of the logical possibilities for the proof's diagram, showing how the respective diagram for each of four cases is related to the proof's text (figure 4.9). Recorde prizes instructiveness, Billingsley completeness.

At the same time, Billingsley's completeness is necessarily a qualified one. Previous editions of the *Elements* in other languages break proofs down by

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<sup>39</sup> Sig.[fist]2<sup>v</sup>. <sup>40</sup> This feature can also be found, albeit less frequently, as far back as the 1482 Ratdolt text. <sup>41</sup> Fol.10<sup>r</sup>–11<sup>r</sup>.

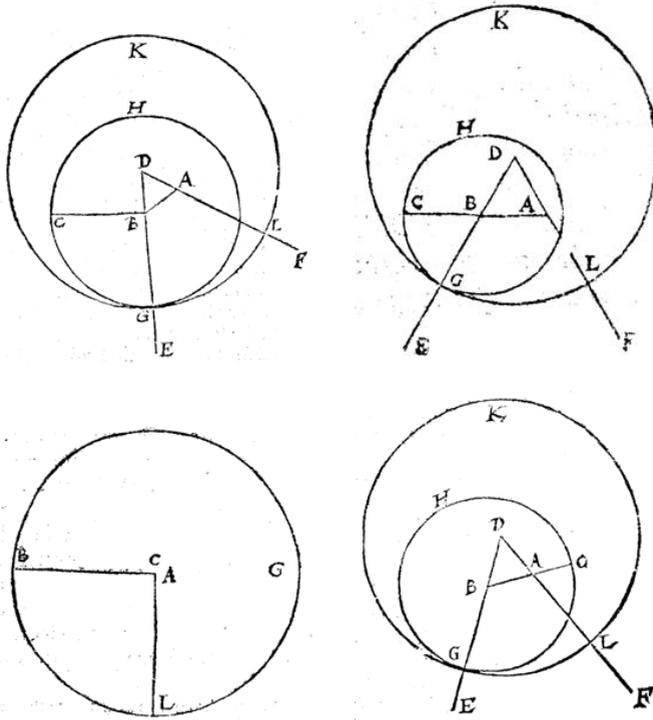


Fig. 4.9: Billingsley's (1570) four cases for Proposition 2. Fol.10<sup>v</sup>-11<sup>r</sup>

their possible diagrams in similar ways, but Billingsley appears particularly zealous in treating proposition I.2 in this manner. Just as Recorde uses demonstrations by simple manual practices such as drawing early in his text where such demonstrations are still simple and plausible, Billingsley can only afford to be exhaustive with such proofs at a relatively early stage. Thus, Billingsley's detailed demonstration of proposition I.2 manages to stand in for the great range of demonstrations where such a consideration would be prohibitively impractical. He shows how one diagram and argument can stand for many in this simple case so as to avoid having to do so for later ones.

Recorde and Billingsley's different approaches to instructiveness and completeness play out dramatically in the different illustrations they attach to Euclid's common notions. Where Recorde instantiates the principle with a specific example which illustrates and justifies the claim (as in figure 4.6 above), Billingsley aspires to the most abstract possible representation by

joining the claim to images of appropriately related line segments (figure 4.10). Here, Billingsley appropriates a convention found in many prior Euclidean texts from later books of the *Elements* where quantities are depicted as linear magnitudes, often arranged in series next to each other for ease of comparison. The segments become, for Billingsley, standard representatives for any type of quantity, and could just as well be of any length or dimension. In Billingsley's metonymy of magnitude, "a line, which is the first kynde of quantitie,"<sup>42</sup> stands in for all geometric quantities. The letters labeling their lengths reinforce this point, as well as the convention in geometric diagrams of using such labels to produce general geometric arguments. Recorde's rectangles, on the other hand, work only as rectangles and have specific numerical sizes associated to them.

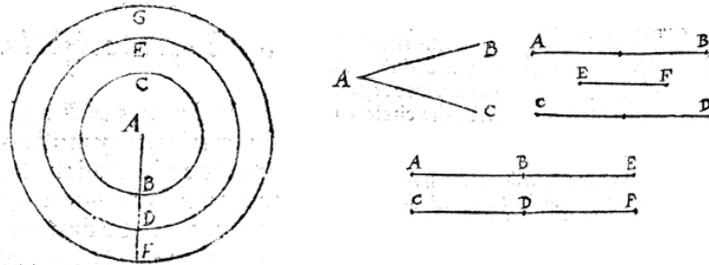


Fig. 4.10: Illustrations of postulates and common notions from Billingsley (1570) Fol.6<sup>r</sup>-7<sup>r</sup>

Similarly, Recorde depicts a compass to show how circles of arbitrary center and radius may be made (figure 4.4 above), whereas Billingsley asserts their multiplicity by drawing a nested collection of three circles with a common center described along a single radial line (figure 4.10). Where Recorde shows how the postulated circles can be produced, it is enough for Billingsley to assert that they can be. Like Recorde (figure 4.8 above), Billingsley shows two straight lines failing to make a surface (figure 4.10), but does not show the feat being accomplished when lines are allowed to bend. As before, it is a simple fact in Billingsley's presentation that straight lines cannot enclose a surface; he does not strive like Recorde to graphically detail the scope and import of the claim as he makes it.

The same disanalogy applies to the theorems of the two works. Billingsley's and Recorde's illustrations for Euclid's theorem that a pair of circles can cross at most twice<sup>43</sup> both depict a circle crossed four times by an eye shape,

<sup>42</sup> Fol.1<sup>v</sup>. <sup>43</sup> Theorem *lv* (Sig. *i2<sup>r</sup>*) in Recorde, book 3 proposition 10 (Fol.89<sup>r</sup>) in Billingsley.

the standard figure (with very few exceptions) for this proof in the Euclidean canon. This non-example provides a starting point for a proof by contradiction which is spelled out in Billingsley's translation but only hinted at by Recorde. Recorde, however, also includes another circle of the same size to show how circles do indeed cross,<sup>44</sup> as well as an ovular 'tunne forme' to show, along with the eye form, that only 'irregulare formes' may violate the theorem. He thus adds a surplus of pedagogic detail to facilitate understanding of the range of the theorem's implications. Billingsley's figure is an accessory to the proof of the theorem, never seeking to show more than the relationships between different objects cited in the proof and providing a means of visualizing the series of letters and shapes to which the textual demonstration refers. A similar contrast in approaches is present throughout Recorde's and Billingsley's works, reaching all the way back to the definitions.

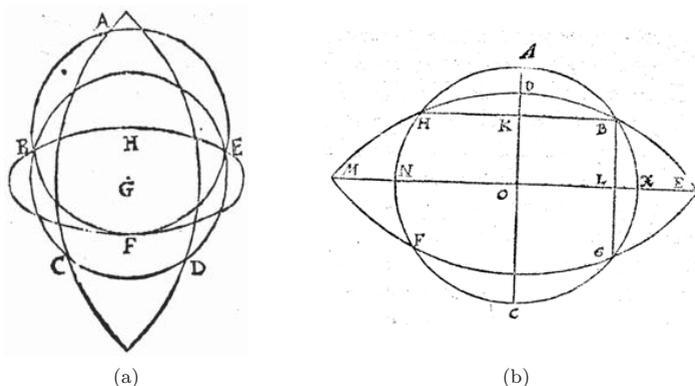


Fig. 4.11: Proofs about intersecting circles: (a) Recorde (1551, Sig.i.2<sup>r</sup>), (b) Billingsley (1570, Fol.89<sup>r</sup>–89<sup>v</sup>)

This is not to say that Billingsley wholly disregards pedagogic considerations or indications of how geometry might look in practice. Both Recorde and Billingsley include figures demonstrating how compass marks might economically be produced in service of a construction (figure 4.12). In Recorde's case, the construction is a practical non-rigorous shortcut. For Billingsley, on the other hand, the figures showing compass marks indicate how only certain arcs of circles need be drawn in order 'readily' to produce triangles in good Euclidean form – they are not allowed to stand in for the thoroughly Euclidean constructions in later proofs.

<sup>44</sup> Pacioli (1523) shows only two circles crossing at two points for this theorem.

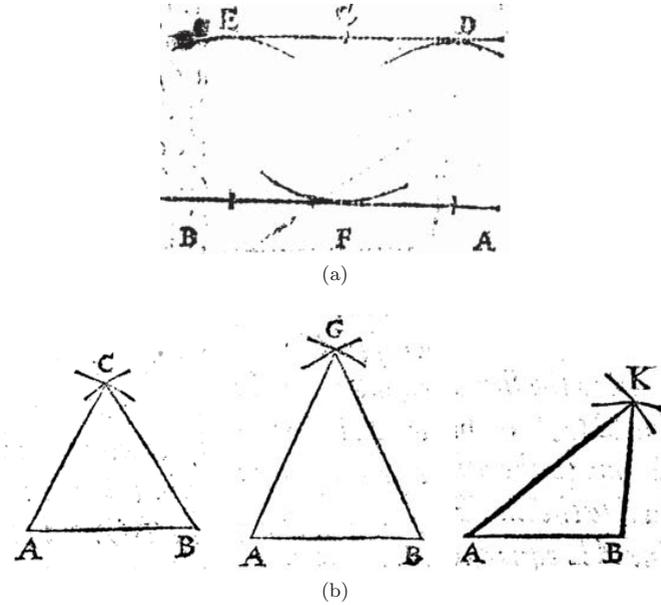


Fig. 4.12: Illustrations involving compass marks: (a) Recorde (1551, Sig.*D2<sup>r</sup>*), (b) Billingsley (1570, Fol.10<sup>r</sup>, triangles arranged vertically in original)

The comparison between Recorde and Billingsley takes another dimension in their diagrams for the Pythagorean theorem, relating the lengths of the sides of a right triangle. Recorde builds his figure from a right triangle whose sides are in the ratio of 3-4-5, dividing each side and its associated square accordingly. He writes that “by the nombre of the divisions in eche of these squares, may you perceave not onely what the square of any line is called, but also that the theoreme is true, and expressed plainly bothe by lines and nombre.”<sup>45</sup> Because his aim is to illustrate the theorem in as comprehensible and multifarious a way as possible, Recorde depicts a right triangle with the simplest combination of sides whose lengths are related by ordinary ratios of integers. This allows him to make the demonstration “bothe by lines and nombre” in a readily graspable format, and his textual explanation is a step-by-step description identifying the features of the figure with the general claims of the proposition – first identifying the shapes, then showing how to read their respective areas from the diagram, and finally affirming that the

<sup>45</sup> Sig.*e4<sup>r</sup>*.

proposition is satisfied in the depicted case before describing how to use the proposition to find unknown sides for other right triangles.

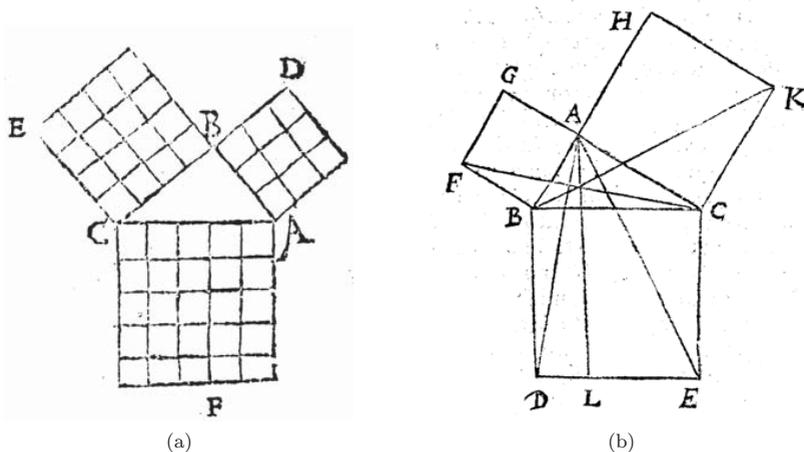


Fig. 4.13: Diagrams for the Pythagorean theorem: (a) Recorde (1551, Sig.e4<sup>r</sup>), (b) Billingsley (1570, Fol.58<sup>r</sup>)

This figure is a partial exception to Recorde's rule of adding extra details to his figures for those who would learn their conventional proofs. The standard Euclidean proof, corresponding to Billingsley's diagram, requires a number of auxiliary lines to allow the areas of the squares to be compared by a means other than counting unit squares – something which would not even be possible with the triangle in Billingsley's figure because his sides and hypotenuse do not appear to correspond to any simple Pythagorean triple of integers when measured. Recorde's diagram, however, manages to invoke its Euclidean counterpart. The orientation of the triangles and squares is an obvious parallel. Easier to miss, Recorde labels a point 'F' at the bottom of his diagram which corresponds not to any of the corners or crossings of his figure but to the point labeled as 'L' from the vertical auxiliary line in Billingsley's figure.

In the case of the Pythagorean theorem, as, indeed, with most of Euclid's propositions, Recorde's text can hardly be construed to provide even the outline of a conventionally rigorous argument. Recorde's gestures at this distant rigor – the 'F' label, the extra lines in other diagrams – point rather to an ideal of what geometry is and what it is about. *Pathway's* readers did not learn to prove, but they saw what proofs looked like and, perhaps more importantly,

they saw what proofs could show. Recorde's is thus manifestly a geometry of showing and, insofar as it was practical for his vernacular readers, of doing as well.

Billingsley's geometry, like Recorde's, aims to explain the meaning and value of both geometry's results and practices. Unlike Recorde, however, Billingsley insists on doing geometry even when it is not a simple matter of filling in a dotted line or hanging a ball of lead from an archway. In this sense, Billingsley's text appears to us as a work *of* geometry, while Recorde's seems more *about* geometry. This reflects, in large part, the different pedagogical approaches taken by the two translators. But it also reveals a bias I would like to suggest is distinctly posterior to these writings.

For both Recorde and Billingsley, proofs are essential to geometric knowledge. Recorde, however, presents a geometry in which man's senses and actions are prior even to the proofs. Focusing on the results and applications of Euclidean geometry as they are available to perception, *Pathway* need not be seen as deficient for lacking the sort of rigor later imagined as the heart of the geometric method. Rather, Recorde's geometry treats first things first: the sensible takes priority over the rational throughout the book, just as it did in Recorde's definitions. Billingsley, then, departs from Recorde only insofar as he gradually allows the rational to assert itself where the senses do not suffice. This is not to argue that this one contrast need overthrow our present received view of Early Modern Euclidean rigor, but to suggest that other readings are possible, and indeed may account for some features of texts that might otherwise pass without notice.

Before outlining the dramatically different illustrative strategies in Digges's *Pantometria*, a few words on those of Dee's preface are in order. Indeed, Dee's preface is striking for its lack of geometric illustrations, and contains only three small diagrams relating to geometry's applications and his famous taxonomic diagram of the mathematical arts and sciences, despite treating several concepts related to some which Billingsley finds necessary to illustrate extensively. Dee's text thus stands as an important corrective for the comparisons undertaken in this article, a role which will become more explicit when all three volumes are directly compared at the end. For Dee, the relationships between mathematical objects and sciences are to be understood logically and schematically, not diagrammatically. Even the diagrams he does use, including the fold-out diagram at the end of his preface, emphasize that it is the order of the mathematical sciences which is at the heart of his work, not the understanding of their constituents.

## 4.5 Digges's Geometry in Context

Like Dee's preface, Thomas Digges's appended discourse on geometric solids is only sparsely illustrated. His definitions include canonical projections of Pla-

tonic solids along with ornately lettered name labels (figure 4.14) and some other geometric projections and line diagrams for complicated constructions and calculations appear later. But there is little to indicate that he intends readers to understand his results with the help of in-text diagrams. As a compendium of new mathematical results, Digges's discourse is the least pedagogical of the texts here considered, and its transmission of Euclid is more by way of form and style than textual content or imagery. Thus, he begins with definitions and presents results in the form of Euclidean propositions, using Euclidean terminology and rhetoric throughout.

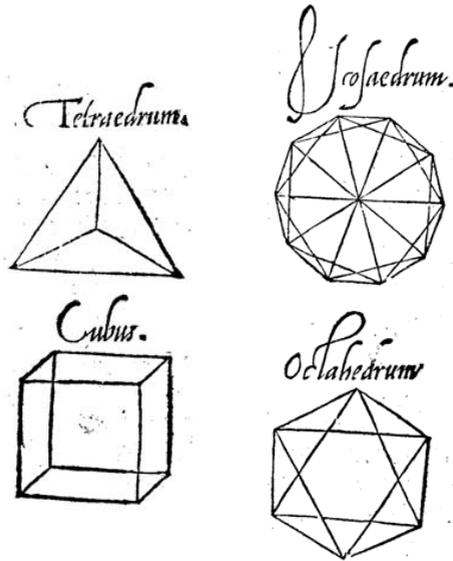


Fig. 4.14: Images of polyhedra from Thomas Digges's (1571, discourse, Sig.T2<sup>r</sup>-T2<sup>v</sup>)

The *Pantometria*, on the other hand, is richly illustrated with conventional geometric figures and examples, plans for surveying instruments, and, above all, detailed scenes of geometry in practice. Leonard Digges's definitions, like Recorde's, are illustrated and labeled in a way that allows them to stand on their own without textual explanations. Indeed, the language of Digges's definitions is so spare and unelaborated in comparison with the rest of the text and with Billingsley and Recorde's renderings that it can easily be seen as secondary to the diagrams, a perfunctory nod to the norms of Euclidean exposition. Without much aid from their surrounding text, the diagrams of

the *Pantometria* systematically illustrate geometric concepts and their relationships. They do not, like Recorde's, aim to show the wide variety of objects under consideration, but rather depict each concept in a single case in order to establish a working vocabulary.

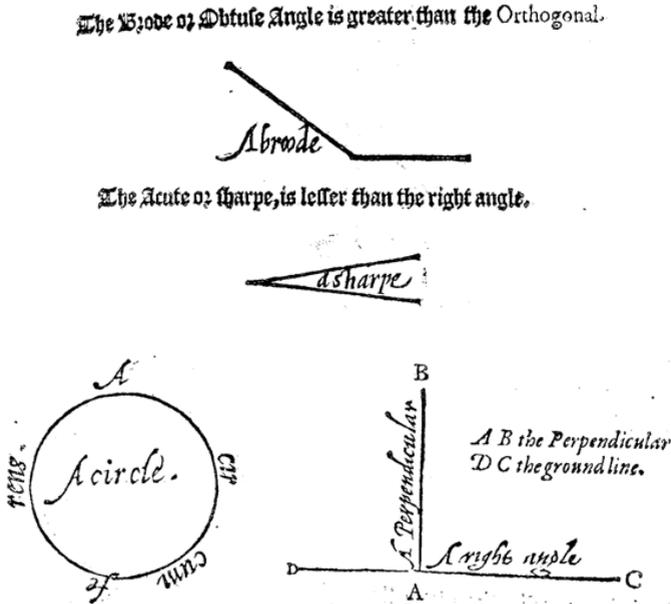


Fig. 4.15: Definitional figures from Digges (1571, Sig.B1<sup>v</sup>–B2<sup>r</sup>)

These single cases, however, are not portrayed in isolation. Even Digges's simplest definitional images are arranged in what might be called 'conceptual scenes' which show how his concepts are related. Thus, a point appears alongside two types of lines, and terms related to angles, circles, or perpendicular lines are joined in single composite images (figure 4.15 and figure 4.28 below). The diagrams establish a touchpoint for a new geometric vocabulary, and help the reader to systematize the large variety of new definitions by visually associating related terms and images.

The geometric concepts of the definitions are then given contextual meaning within elaborate scenes of surveying and warfare. These scenes impose geometric lines, measures, and instruments on landscapes and in settings where they might be used. They often contain additional buildings, people, statues, and decorations, in many cases significantly more stylized than the ob-

jects most closely implicated in the geometry under consideration, in order to establish the setting.<sup>46</sup> In some cases, geometric measurements are made by surveyors (figure 4.16 shows a surveyor making three measurements over time), but in other scenes geometric features are superimposed on the landscape without the benefit of an instrument, observer, or either of the two (e.g. figure 4.17). Additional people in the scenes help stage the measurement either directly, as where a finely attired woman gestures at the surveyor (figure 4.18), or indirectly, as where armies stand and wait for the geometer to finish his work (figure 4.19) or hunting parties chase game which is perhaps to be served in the hall being measured (figure 4.20). The latter includes hunting parties in both the foreground and background, corresponding to surveyors at either end of the hall.



Fig. 4.16: Scene with time-lapse measurements from Digges (1571, Sig.D1<sup>r</sup>)

The difficult work of bridging representational conventions in landscape art and geometry often creates striking oddities in the scenes. The figures from the text use a standard repertoire of techniques to establish depth and perspective, but these techniques are not applied to the geometric figures overlaying the landscapes. The result is that where the geometric figure itself has depth (that is, when it is not in the plane perpendicular to the viewer's line of sight) there is a visible incongruity with between the geometry and its scene. Attempting

<sup>46</sup> Many of these visual features can be found in, for instance, Pacioli 1523, Fine 1544, or Frisius 1557.



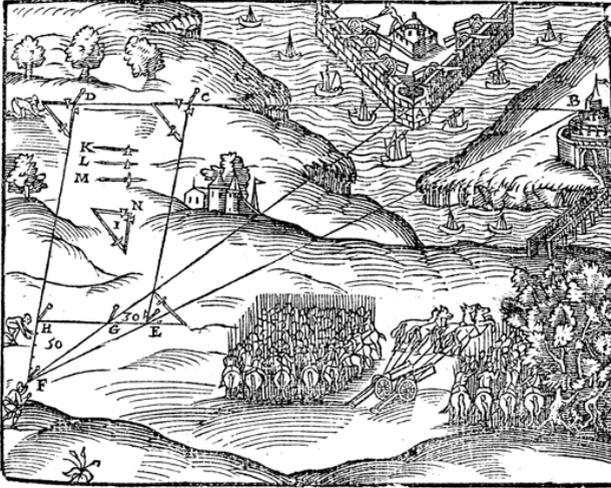


Fig. 4.19: A military scene from Digges (1571, Sig.F2<sup>r</sup>)

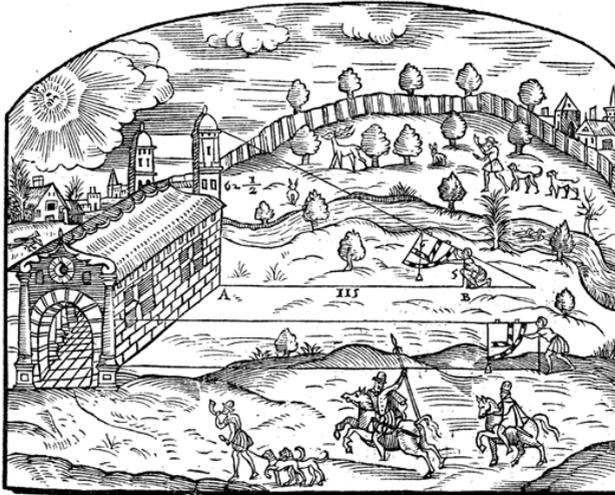


Fig. 4.20: Scene with hunters and a hall from Digges (1571, Sig.D4<sup>r</sup>)

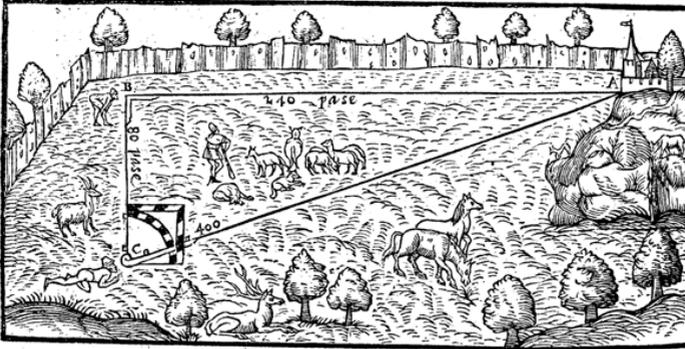


Fig. 4.21: Surveying a pasture, from Digges (1571, Sig.G4<sup>v</sup>)

A similar tension emerges where depth is created by other means than perspective. Figure 4.22 depicts the determination of the “true water levell from a fountaine.”<sup>47</sup> Digges makes a fountain’s height plausible with a winding path, but the depth associated with the path vanishes in the fountain’s geometrization. In scenes such as this, geometry is not necessarily made visually realistic, but rather is given a situational context where the geometer’s figures and the viewer’s scenes can comfortably (if not always naturally) coincide. More broadly, distortions of scale and other visual simplifications or embellishments in the *Pantometria* give rise to scenes which do not precisely depict actual users of geometry in their past or anticipated work. Rather, they conjure a constellation of images which appeal to geometry as a practical, worldly, and even glamorous endeavor and reinforce the plausibility of both the geometric methods themselves and their purported applications.

Nothing better represents the vexed nexus of geometry and familiar experience than the appearance of instruments in the *Pantometria*. Geometry, after all, was wholly alien to the work’s vernacular readers, and was made less so by association with familiar scenes and contexts. Measuring and surveying instruments, on the other hand, are not nearly so otherworldly as the geometric entities they help to produce. They are real objects and readers may indeed have seen them without knowing their full role in geometry, but the work and its illustrations are also premised on the presumption that such instruments be also unfamiliar and outside of the normal experience, in both form and use, of Digges’s audience.

Within the *Pantometria*, geometric instruments play a number of roles. In the work’s many scenes, the instruments work both to establish geomet-

<sup>47</sup> Digges (1571, Sig.K1<sup>v</sup>).

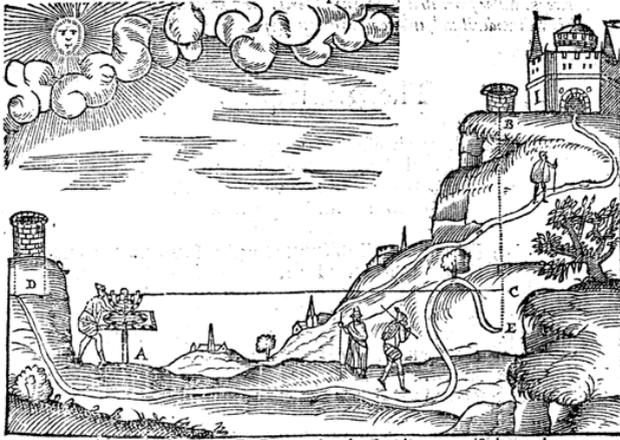


Fig. 4.22: Geometric scene with a fountain on a hill from Digges (1571, Sig.K2<sup>r</sup>). (Here, ‘fountain’ is synonymous with ‘well’)

ric properties and show their manners of measurement. Thus, a draftsman’s square produces right angles in two related ways: it shows which angles in a scene are right angles by virtue of their association with the instrument and it shows how such angles can be produced by the surveyor who would apply the lessons of the construction or calculation from the scene. Embedded quadrants have a similar function for non-right angles, and lengths are shown with regularly spaced marks along lines or with labels indicating a certain number of paces.

In this way, scenes show more than just contexts for the geometry of the *Pantometria*. They also use depictions of instruments within those contexts to bridge the scenes of the work and the sites of the work’s potential application. Showing instruments in use, Digges also shows how they are to be used. This principle holds in the scenes discussed above, which depict instruments alongside their users, but it also applies where instruments are shown free of the surveyors or geometers who might use them. In figure 4.19, for instance, Digges shows how to produce an angle using “three staves, halberdes, billes, or any such like things, K L M”<sup>48</sup> and depicts several such arrangements, both on their own and being used by a surveyor. In figure 4.23, the geometric tools are given full-name labels within the scene and are shown performing a simple geometric measurement on their own, independent of the geometer’s interventions.

<sup>48</sup> Sig.F1<sup>v</sup>.

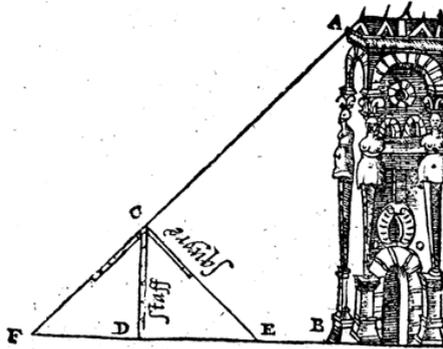


Fig. 4.23: Geometric scene with labeled instruments from Digges (1571, Sig.E2<sup>r</sup>)

In some places, the line between geometric instrument and geometric object is blurred in Digges's diagrams. He refers, for instance, to smaller triangles which one could construct and measure as a replacement for cumbersome trigonometric calculations. These triangles are depicted within the larger scene in order to show all the geometer's resources in one and the same image, but appear out of scale so that their construction and dimensions can be more legibly rendered. Figure 4.24 has one such triangle with whose aid the geometer in the example measures a much larger similar triangle. Here, the auxiliary triangle floats somewhat apart from the scene, away from its users and in an otherwise unfilled part of the landscape. It is a necessary part of the calculation, but it could, implies Digges, be anywhere, at any scale. Artistic convenience here coincides with mathematical principles about similar triangles and their use in calculations across large scales. If the auxiliary figure is particularly complex or its associated context particularly difficult to depict, it might even be shown alone as a stand-in for a more detailed geometric scene, as is the case for the construction in figure 4.25 for an example involving the determination of distances between landmarks.

Measuring instruments themselves also appear in isolation and with considerable detail in the *Pantometria*. Illustrations such as those of figure 4.26 would have taught readers how to imagine the details of the coarsely schematized instruments in Digges's scenes as well as how to build such instruments for themselves. From the details of the images, one can discern something of the instrument's materials, features, and even assembly. The images accompany written instructions which guide the reader through each instrument's production, along with some indications regarding its use. *Pantometria* was, after all, a text of practical and applied geometry. In order to make Euclidean



Fig. 4.24: Scene with an auxiliary triangle from Digges (1571, Sig.G3<sup>v</sup>)

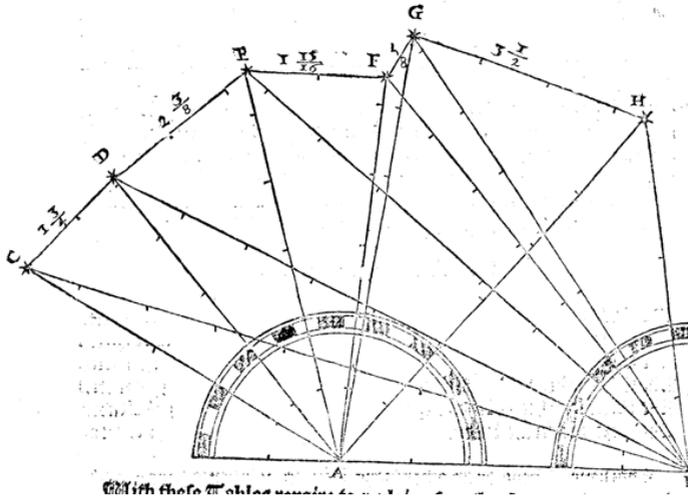


Fig. 4.25: Geometric figure for an extended computation from Digges (1571, Sig.K4<sup>v</sup>)

geometry relevant to the *Pantometria*'s readers, the work had to fill in the gaps between Euclidean ideals and geometric experience. Instrument-making was, for these users of geometry, one of the most essential mechanisms for this gap-filling.

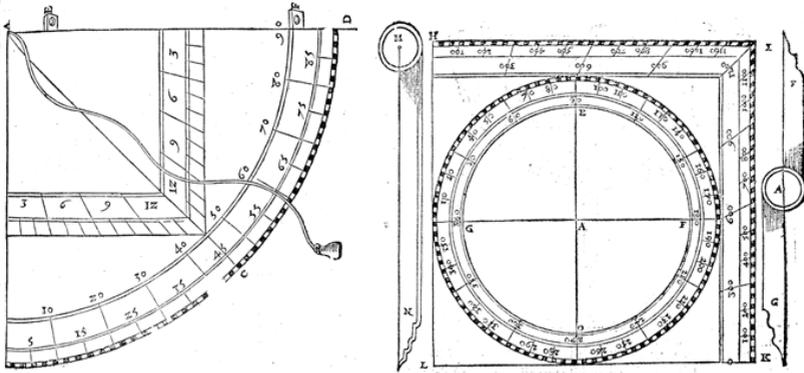


Fig. 4.26: A quadrant and a theodolite from Digges (1571, Sig.C4<sup>r</sup>, I2<sup>r</sup>)

## 4.6 Points and Parallels

Two examples from the geometric definitions in the works under consideration help to highlight different authors' contrasting approaches to geometric figures. Before considering their illustrations of the geometric point, it will help to examine a slightly more complicated notion: that of parallel lines. Recorde, Billingsley, and Leonard Digges each include an illustration for the concept of a parallel line (figure 4.27). From the rich history of the parallel postulate in Euclidean geometry it should be clear that what it means for two lines to be parallel is by no means self-evident. Staging parallel lines with different sorts of diagrams, the authors bring to the fore different aspects of the 'parallel' concept and different roles parallel lines play in Euclidean geometry.

Digges provides the most straightforward image of parallel lines. His figure depicts two horizontal line segments of the same length framing the caption 'Paralleles.' As with Digges's other definitional figures, this one serves to establish an operating vocabulary. The two depicted lines are parallels, and act as a point of reference for future invocations of the parallel concept without being exhaustive of all possibilities for its manifestation. Digges defines parallel lines as ones "so equedistantly placed"<sup>49</sup> that they never meet, and underscores this notion by making his example lines not just equidistantly placed but also of the same length. Parallel lines, according to Digges, are characterized by their levelness and their equalness, a message reinforced by the spare details of his image.

<sup>49</sup> Sig.B4<sup>r</sup>. Equidistance seems an unusual criterion to modern readers, but is not hard to find in Early Modern texts. In addition to Digges and Recorde, their contemporary Petrus

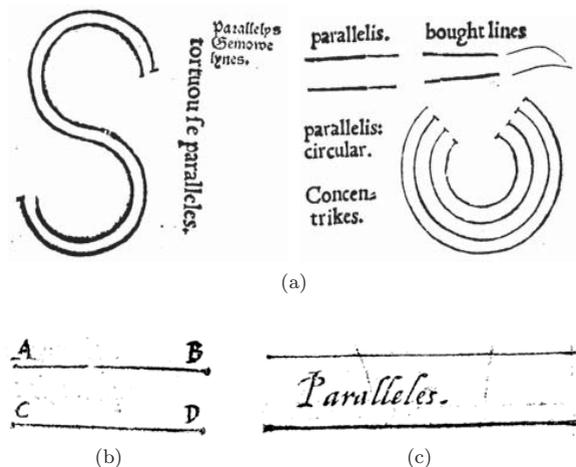


Fig. 4.27: Figures for parallel lines: (a) Recorde (1551, Sig.A4<sup>r</sup>–A4<sup>v</sup>), (b) Billingsley (1570, Fol.5<sup>v</sup>), (c) Digges (1571, Sig.B4<sup>r</sup>)

Parallel lines for Billingsley have a very similar image as for Digges: he too uses evenly spaced horizontal segments of the same length. But where Digges labels his with a full word, Billingsley marks his parallel lines with four letters placed at the endpoints of the two segments. In Billingsley's text, the system of reference established by the letters – that of referring to lines by pairs of points therein – is more important than Digges's crude taxonomic nomenclature. That the lines are parallel is something to be proved or stipulated in the text with the help of labels and not something to be observed from the ostensible properties of their appearance on the page. Billingsley thus uses the labeling scheme for his figure to displace the property of parallelism into the written text, even as his image reproduces the same conceptual shorthands – evenness, levelness, equalness – as does Digges's.

Recorde also has an image of two horizontal segments with the label 'parallelis', but this is just the second of four labeled examples used to illustrate the concept. On the page where he introduces the parallel concept, the image is of two parallel S-shaped curves. In Recorde's textual definition, equal spacing is the paramount feature of parallel lines, and the 'tortuose paralleles' of his first figure emphasize this point by showing that no matter where a line turns its parallel must turn with it in order to stay evenly spaced. As suggested above, Recorde's definitions deal far more in curved lines than do

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Ramus (1580, 14) has equidistance as his explicit definition for parallel lines.

the subsequent parts of his text, and his definition for parallel lines is no exception. Curves allow Recorde economically to embed geometric multiplicity in relatively few figures. A single S-curve shows how parallel lines work in every direction and under any transformation.

Also in keeping with other definitions, Recorde illustrates the parallel concept contrastively by showing two pairs of non-parallel lines next to his canonical horizontal parallels. This pairing has two functions. First, it shows just how parallel lines differ from non-parallels. Even though Recorde's non-examples do not cross, one can see very clearly where they will meet by following the courses of the paired lines from left to right. Recorde thus provides a visual gloss to aid in recognizing parallelism and non-parallelism in figures where these properties might be ambiguous. But second, Recorde establishes the image of even horizontal lines as the canonical one for the parallel concept. Both by giving it the simplest label – 'paralleis' without any adjectives – and by setting it opposite his contrastive examples, Recorde makes horizontal parallels a default reference point for the concept.

Recorde's parallel concentric circles complete his suite of examples. No longer equal in length, these continue to show the 'equality' aspect of parallelism by covering the same angular scope. Moreover, these have marked end-points in order to identify the appropriate corresponding points for judging even spacing, suggesting how such points might be used to gauge parallelism more generally. They show yet another way to judge parallelness, and emphasize that equal length is not the only possible clue indicating the parallelness of two lines. Recorde thus closes his illustration of the parallel concept by insisting, as he does throughout his definitions, on the concept's multiplicity and wide scope.

Unlike parallel lines, the simple geometric point appears at first to be an unproblematic concept whose properties are largely self-evident from the common experience of any potential reader. Perhaps because it is the first defined object in the Euclidean corpus, the point receives a level of attention seemingly out of proportion to its obvious simplicity. All but Thomas Digges offer definitions. Of those, all but the elder Digges add explanatory notes and all but Dee offer illustrations (figure 4.28). The challenge, for our authors, was to establish a relationship between the points of common knowledge and the geometric points of the Euclidean texts. For our authors' readership, points were recognizable commodities whose manifestation in geometry was nonetheless completely alien. Textual and visual cues conspire to transfigure common points into Euclidean points, and thereby to set each work on a suitably rigorous Euclidean foundation. There is not much to distinguish the geometric points' visual manifestation in the works under consideration, but the subtle differences that do exist become quite stark and significant upon consideration of their textual context.

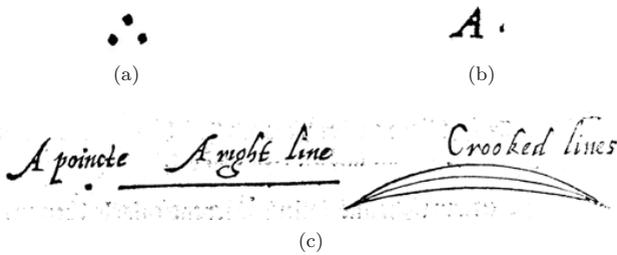


Fig. 4.28: Figures for the geometric point: (a) Recorde (1551, Sig.A1<sup>r</sup>), (b) Billingsley (1570, Fol.1<sup>r</sup>), (c) Digges (1571, Sig.B1<sup>r</sup>)

Billingsley illustrates the geometric point with a small dot in the outside margin to the right of a label ‘A’, which is printed slightly larger than the labels in Billingsley’s other geometric diagrams. Digges shows a large dot labeled ‘A pointe’ arrayed horizontally next to a similarly labeled right line and collection of crooked lines in a figure below his third definition. Recorde’s three exemplary points, placed at the end of his paragraph, are about the size of his punctuation marks and are arranged in a small upright equilateral triangle. Although Dee does not include a drawing of a point, he does connect Euclidean points to visual experience by explaining that “by visible formes, we are holpen to imagine, what our Line Mathematicall, is. What our Point, is.”<sup>50</sup>

Indeed, for both Dee and Billingsley there is an explicit call for the reader’s imagination to make the final leap from visual to geometric points. Billingsley calls a point “the least thing that by minde and understanding can be imagined and conceived: then which, there can be nothing lesse, as the point A in the margent.”<sup>51</sup> The printed mark in the margin is small, but it is surely not the least thing imaginable. Rather, Billingsley’s figure indicates the relevant features of points for his exposition, including their smallness and amenity to labeling by a single letter. Billingsley’s explanation instructs the reader to regard his textual model as the least thing imaginable, with the letter next to it understood as the point’s name.

Dee’s and Digges’s use of point-figures sit at two possible extremes. Dee, on the one hand, does not illustrate points at all. Not wishing to build his preface on Euclidean diagrams, he has no need for the visual geometric literacy so necessary for following the others’ expositions. Points are pointedly not illustrated. It is the point’s (textually manifested) philosophical relation to other

<sup>50</sup> Sig.a2<sup>r</sup>. <sup>51</sup> Fol.1<sup>r</sup>.

objects and ideas which is important to Dee, not its operational centrality in geometric proofs and figures. Dee's words are rich and elaborate, placing points in a broader schema for all of mathematics.

Digges, by contrast, bluntly and without elaboration states that "A *Point* I call whiche cannot be divided, whose parte is nothing."<sup>52</sup> His definition would have been of little use to *Pantometria*'s readers. Instead, their information about points came primarily from his diagram, which shows a representative point and indicates its relation to lines. The definitions for lines and right lines, which explicitly describe lines as having points for their extremities, combine with the point's definition and depiction to guide the reader to imagine points as the indivisible bounds of short segments of lines. This operational understanding covers Digges's uses of points, for which the 'whose parte is nothing' aspect plays no formal role.

Recorde, finally, places his points within a geometric arrangement. Even when they are the sole subject of the illustration, his points participate in a larger geometric context. Moreover, Recorde's triangle of points, like the text that accompanies them, emphasizes more than any of the other images how truly common the geometric point should be to his readers. He stages his points amidst punctuation marks and descriptions of pen pricks so as to establish their meaning in the familiar contexts of writing and reading. As his first geometric illustration, the points in Recorde's figure begin the difficult work of bridging everyday experience and geometry by showing how geometric texts produce meaning through arrangements of familiar forms.

## 4.7 Conclusion

The first heralds of English geometry – Recorde, Billingsley, Dee, and Leonard and Thomas Digges – produced, over a twenty year period, three starkly different geometric texts. Their attempts to translate Euclid's *Elements* into vernacular English brought with them an opportunity to reimagine the whole of geometry for a new audience. As the comparisons in this essay indicate, visual images figured centrally in this reimagining. Our authors used geometric figures and diagrams to show geometry vividly to their readers, and their different strategies of illustration place different emphases and establish different priorities in the English geometries they aimed to create.

All translations involve the attempt to convey meaning from one idiom and context to another. It is axiomatic that a translator is faced with a wide range of textual considerations which can dramatically affect the meaning of the resulting work. Nor is it surprising that similar considerations play out in the

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<sup>52</sup> Sig.B1<sup>r</sup>.

non-textual elements of the translation. More than just decorations or elaborations, diagrams in geometric texts are crucial vehicles for both meaning and context. The diagrams in the first English geometries irrefutably participated in those geometries' construction.

For all the ink that has been spilled in the analysis of the *Elements* and for all the comparatively few analyses of the neologisms and other non-diagrammatic features of its first English translations, much remains to be learned from the role diagrams play in Euclid's re-renderings. Studying diagrams under translation, like the corresponding study of the translation of words and phrases, can say a great deal about the work of interest. It can show how the work was received, what it meant to its translators and users, and also what is possible in the work's interpretation and transmission.

Studies such as this one open the way for a richer discussion of the purpose and function of Euclidean diagrams in general. They show, for instance, what features of Euclidean diagrams were considered important, by whom and for whom. They show how the relationships between diagrams and their textual context change over time and between audiences. They show, moreover, how a simple geometry and its associated visual tradition maintained, at least for the Early Moderns, a remarkable level of interpretive and representational flexibility.

As Euclid crossed into a new tongue, his translators each refracted different features of his geometry. By contrasting their diagrams, I argue, we may better glimpse the contrasts between the Euclidean translators, and ultimately gain a better insight into what it means to translate Euclid.

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Abbreviations: CamUL = Cambridge University Library; CUKL = Cornell University Kroch Library, Rare Books & Manuscripts; CULAC = Cornell University Library Adelman Collection; CULHC = Cornell University Library Hollister Collection; CULHSC = Cornell University Library History of

Science Collection; DNB = Oxford Dictionary of National Biography, online, Oxford University Press; HL = Huntington Library, online access via Early English Books Online; OBL = Bodleian Library, online access via Early English Books Online; SJCL = St. John's College Library, Cambridge; STC = Short Title Catalogue, 2nd edition; Yale University Library, online access via Early English Books Online.

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