“That which hath no part”:
The Euclidean Point at the Dawn of English Geometry

Michael J. Barany, Princeton University

Abstract

The surprising diversity of definitions of the Euclidean point from the first century of English vernacular geometry raises a pair of intertwined problems in the historiography of logical knowledge: first, that beneath the formal unity of such knowledge there tends to be a great diversity of interpretations; and, second, that this diversity does not generally prevent the appearance of unity in the objects and inferences of such knowledge. I propose that this diversity of interpretation, when read in the context of textual conventions that reinforce formal similarities, helps authors to ignore divergent fundamental interpretations as inessential. Only when foundations become argumentative resources, rather than practical or pedagogical ones, do such consensual elisions break down. For the English point, authors’ varied goals, personal connections, and relations to respected authorities permitted a variety of interconnected formulations of puncticity that underscore the heterogeneity latent in the history of fundamental knowledge.

Introduction

A *point* is that which has no part. (Heath 1956, 155)

In English, eight short words in four iambs begin one of the most important works in the history of science. Euclid’s *Elements* looms large both for its particular contents and for the centuries of epoch-defining debates and interpretations it has nucleated in and beyond the mathematical sciences.[1] It has been scrutinised from head to tail by geometers, philosophers, philologists, historians, and many others.

The familiarity of the iconic opening definition of the Euclidean point makes it easy to look past two complementary phenomena that I shall suggest present crucial problems for historians of mathematics and science. First, the point’s short and scarcely-varying formal definition tends to obscure a great diversity of historical paratexts expressing a significant plurality of interpretations and understandings. Second, such interpretive variation nevertheless rarely has rarely formed an obstacle to a certain consensus (or perhaps impasse) about the ultimate status of the Euclidean point. Though its rhetorical formula has been subject to manifestly divergent interpretations, most authors have seemed to agree that there is just one such thing as a Euclidean point, and that they were all writing about it.

This essay examines the definitions of the Euclidean point in the first printed English translations of the *Elements*, which began to emerge some seven decades after the work's first appearance in print. While there were certainly geometers in England before 1551, when Robert Recorde published his *Pathway to Knowledg*, and while Euclid's *Elements* had by then been firmly established in other languages in English higher education, these vernacular editions of the *Elements* marked the enunciation of what their authors and advocates propounded as a *new* and distinctively *English* geometry (Taylor 1954, 17). This self-consciously fashioned project of English geometry was not just a matter of rendering a unitary text from Greek mathematics in a different language, introducing new terminology like "straight line" (Recorde's coinage) where needed.[2] Rather, as I have argued elsewhere (2010; see also Barrow-Green 2006), it was a multifarious enterprise that reconfigured both English and Euclidean words, idioms, images, ideas, social structures, and practices alike for an audience both imagined as new and in many cases genuinely so.

A synthetic account of the variety of point definitions in English vernacular geometries of this period offers, on the one hand, a useful tracer for comparing these accounts' many narrative, practical, and philosophical features, stakes, and contexts. On the other hand, by widely

---

[1] One can appreciate the range of such debates from Høyrup (1996), Mancosu (1996), and Richards (1988).

[2] Most of Recorde's coinages were less successful, some not surviving even to the end of his book. See

surveying a single vernacular literature, I offer an alternative to the monolithic analyses of a few famous authors or books that can dominate conceptual histories of this period (e.g. Schäffner 2005). Restricting myself to this particular vernacular corpus, I here omit two groups of authors who would nonetheless be known to most of those here considered, and who may well have had discernible influences on the history recounted below. First, there were many famous English mathematical authors who, for various reasons, did not much concern themselves with the Euclidean point. Second, during this same period a range of authors on the European continent made comparable efforts at presenting and interpreting the Euclidean point in other languages. I thus aim not for a definitive history of this particular definition, but rather for a new perspective on that history told with sources not usually assembled in this way, with considerable implications for what is ordinarily taken for granted in histories of early modern mathematics and science.

Instead of decisive innovations or abrupt shifts in perspective, one finds here a variegated field of small divergences and adaptations in authors' choices of words and images. Such a field is shaped broadly by differing approaches to such binarisms as theory and practice or textual authority and authorial intervention, but these commitments are often effaced by more equivocal aspects of the respective geometries. Unremarkable features such as typefaces become, here, notable indicators of authorial and editorial intervention in an open-ended project of translating a canonical sentence that proved remarkably unconstrained by the five Ancient Greek words at the project's supposed core.

If one lesson of this essay is that there is a persistent plurality inherent to even the most apparently simple ideas, the other is that such plurality is itself never immune from historical

---

Johnson (1944) and Johnson and Larkey (1935, 68).

change. Rather than offer an intellectualist history extracted from hypothetical deductions and epiphanies in the long-lost minds of influential thinkers, I here explore what can be said of conceptual change as it happens in a distributed manner across people, texts, and decades, as glimpsed through surviving evidence of historical practices of writing, printing, and reading. What the point lacks in geometrical part or dimension, it gains here in its many dimensions of historical variation and the significant part these can play in the history and historiography of Euclidean mathematics. In this light, the Euclidean point's paradoxical status as at once irreducibly plural and irrefutably foundational makes it a potent point of entry for investigating the narrative articulation of logical knowledge, as such.

A Point of Departure

Among Euclidean objects, the point is exceptional in nearly every way. Euclid's geometry elaborates the properties of figures with magnitudes and many parts, and by definition the point has neither magnitude nor parts. The *Elements* are famed for their deductive method, but, coming first, the point has nothing from which to be deduced. Indeed, even subsequent deductions in the *Elements* invoke the point's definition only implicitly, if at all. But for these same reasons, to understand the *Elements*' generations of translators, editors, interpreters, and users it is particularly helpful to start at the beginning, with the point.

Like other texts of its anitquity, the *Elements* we see today is the descendant of more than two millennia of transmission in multiple languages and across great distances (Rommevaux, Djebbar and Vitrac 2001; Rommevaux 2009; Vitrac 2012). The most influential versions of the *Elements* were, in turn, embedded in commentaries, each with their own attributed authors, interpretive traditions, and variations in transmission that emerged in the curation, adaptation, and improvisation that have constituted the Euclidean corpus over its history. Commentators and translators often felt free to improve Euclidean proofs, modify diagrams, introduce new notations, re-order the propositions, add new theorems, or embellish the text in other ways (Netz 1998). Well before the time of the first English editions of the *Elements*, exposition and reinterpretation were thus standard and significant features of Euclidean texts.

The point's narrative and philosophical primacy in the *Elements* makes its definition a frequent point of departure for such interpretations. Most of the works considered here contained substantial programmatic prefaces, but it was with the point that they put those programs into action. The point's first English vernacular expositors assumed as unfamiliar to their readers both the point as a particular object of geometry and the general form of geometric exposition. At the same time, the Euclidean point was conceptually and visually cognate to several familiar entities, including dots, compass and pen pricks, and the punctuational period (Barany 2012, 152-153). The point was superficially simple, yet could be shown to harbor great depths of significance and even paradox.

Recorde's 1551 translation of the first four books of the *Elements* presents an extreme form of this interpretive heterogeneity, both across the work and in its presentation of the point. His edition liberally re-orders and modifies definitions and propositions; adds, adapts, and omits proofs and constructions; and layers its exposition with frequent examples and didactic digressions. Recorde's opening section on "The definitions of the principles of Geometry" introduces the point not as the *a priori* and *ex nihilo* first object of geometry, but rather through an informal characterisation of all the objects and methods of geometry. Thus:

> Geometry teacheth the drawyng, Measuring and proporcion of figures. but in as muche as no figure can bee drawen, but it muste haue certayne bou[n]des and inclosures of lines: and euery lyne also is begon and ended at some certaine prycke, fyrst it shal be meete to know these smaller partes of euery

> figure, that therby the whole figures may the better bee iudged, and distincte in sonder. (Recorde 1551, Sig.$A1^r$)[3]

Recorde's overture thus inverts the conventional Euclidean presentation of points, lines, and surfaces (including figures). The latter begins with points (definition 1), defines lines (definition 2), stipulates the bounds of lines to be points (definition 3), defines surfaces (definition 5), and then stipulates the bounds of surfaces to be lines (definition 6). Instead, Recorde begins by characterising the practical lessons of geometry in terms of planar figures, then works backwards from definition 6 through definition 3 to establish the point as the primitive unit of geometric study, if not of geometric practice. His inversion proposes that geometric methods proceed by systematically studying figures through their smaller component parts.

Having motivated the point's study, Recorde defines the object as

> *A Poynt or a Prycke,* is named of Geometricians that small and unsensible shape, whiche hath in it no partes, that is to say: nother length, breadth nor depth. (Recorde 1551, Sig.$A1^r$)

The phrase "A Poynt or a Prycke," here transcribed in italics, appears in a larger typeface than the surrounding text in Recorde's book, as do other terms for which Recorde offers definitions.[4] The phrase itself couples the words point and prick, employing a common rhetorical strategy from vernacularizations in this period of pairing a proposed term (here, "Poynt") with a

[3] I have preserved authors' emphases, capitalisations, and non-standardised early modern spellings, expanding contracted characters in square brackets (as in "bou[n]des").

[4] On the same page both the word 'Lyne' and its definition 'Length withoute breadth' are thus typographically differentiated. This and other oddities in the text and diagrams likely arose from a hasty setting from manuscript to print without Recorde's close supervision. Throughout this essay, I refer to the producers of each text under the nominal author's name with the proviso that all these works were the result of multiple people's labour, much of which cannot be disentangled with presently available evidence.

more familiar one (here, "Prycke").[5] Such couplings could also be found in contemporary Latin presentations of the point, and here also reflected a Classical distinction between two notions of puncticity.[6] On the one hand, the point was regarded as a sign that could, for instance, indicate a location in a figure. On the other hand, the point was understood as a puncture, an interpretation Recorde presents as more intuitive and accessible for his readers. The Latin-derived term for this latter conception gives rise to the variant spelling "A poincte" that Recorde places in the margin of his text next to the above-quoted definition.[7]

Recorde's definition implicitly draws on a second split in Classical interpretations, as well. Most authors used the canonical Classical formulation that a point has no parts, but this could mean different things for different authors: either lacking smaller pieces, or (with Recorde) lacking dimensions. Rather than present geometry as a science of (divisible) magnitudes, Recorde presents it in terms of figures in a three-dimensional space. This framing, also present in the paragraph preceding Recorde's definition, matches the point's definition to the ensuing one for a line, which Recorde presents canonically as "Lengthe withoute breadth" (omiting depth).

Having distanced himself from this initial formulation by attributing it to un-named "Geometricians," Recorde then pivots to qualify his definition in practical and sensory terms:

> But as this exactness of definition is more meeter for onlye Theorike speculacion, then for practise and outwarde worke (consideringe that myne intente is to applye all these whole principles to woorke) I thynke meeter for this purpose, to call *a poynt or prycke,* that small printe of penne, pencyle, or other instrumente, which is not moued, nor drawen from his fyrst touche, and therfore hath no notable length nor bredthe: as this example doeth declare.
> ∴

[5] On the use of coupling in vernacularization, see Johnson (1944, 114).

[6] E.g. Hervagius (1558)—a slightly modified re-edition, with Bernhard Brand, of Johann Herwagen's 1537 Basel edition of the *Elements*. The distinction is placed in the context of visual depictions of Euclidean points in Barany (2012, 153).

[7] Cf. figure 1, below.

> Where I have set .iij. prickes, eche of them hauyng both le[n]gth and bredth, thogh it be but smal, and therfore not notable

In place of a theoretical and speculative geometry obsessed with exact definitions, Recorde here offers a conciliatory, practical geometry suitable for his imagined vernacular audience. Though he does not use the common dialogue form found in his other books, he frequently peppers his first-person exposition with formulations like the above (along with second-person exhortations) to set an explicitly pedagogical tone. Recorde's pedagogical geometry, he emphasises, is something done with pens or pencils and concerned only with notable distinctions. While definitional depictions of points were widespread in the sixteenth century, Recorde's inset figure of three points has no counterpart in other printed geometries (Barany 2012, esp. 156). It emphasizes that geometry is visible, produced by hand, and seen in figures rather than in isolated or purely theoretical objects.

Here, Recorde's attestation to having himself made three pricks on the page invokes the act of writing for his readers. This invocation is made explicit in the ensuing sentence, where Recorde introduces the Euclidean line by providing a dotted line on the page and insisting that "if you with your pen will set in more other prickes betweene euerye two of these, then wil it be a lyne" and so makes writing (or at least potential writing) an immediate resource in geometric exegesis (Recorde 1551, Sig.*A*1^{r}).[8]

[8] In the several extant copies of Recorde's book I have examined, no reader seems to have taken him up on this invitation, despite the presence of annotations elsewhere.

A Turning Point

Recorde's successor as the leading proponent of English geometry was surely the brilliant and enigmatic scholar John Dee.[9] When London merchant Henry Billingsley undertook to prepare a lavish English edition of the whole of the *Elements*, he turned to Dee as an ideal authority figure to provide a preface for the volume, which was completed in 1570. Their work marked a turning point for the vernacularization of Euclidean geometry, quickly becoming its era's definitive English translation and interpretation of the *Elements*. In Dee's preface, one encounters a rather different definition from Recorde's:

> A *Point*, is a thing *Mathematicall*, indiuisible, which may haue a certayne determined situation. (Billingsley 1570, Sig.*1^{r})

Dee is unique among sixteenth-century English expositors in making position a defining property of points, though Billingsley and Recorde both remark on this aspect while explaining their definitions. Like Recorde, Dee leads into his definition by descending from multiple dimensions to none. He describes solid bodies in three dimensions, followed by surfaces with two dimensions, then lines that have only length, and finally points, which are the two ends of a line. Unlike Recorde, who stresses the exigencies of drawing figures, Dee presents this sequence with a philosophical emphasis on the dimensional properties that characterize general kinds of "thing *Mathematicall*." Such things, for Dee, can be either numbers or magnitudes, where magnitudes are "long, broade, or thicke" and are divisible.

Dee's point, of course, is neither long, broad, nor thick, nor is it divisible. Just as Recorde's descending series of objects justifies the practically misfit point's inclusion in his

---

[9] Dee's scholarly output and influences are among the most studied of this period. See Rampling (2011) and Roberts and Watson (1990). Extant catalogues of Dee's library include Euclidean geometries

system, Dee's corresponding series offers a philosophical justification for the point's inclusion among mathematical things. That is, "*Terminatiuely*, we reckin it a thing *Mathematicall*" because it is the "bound of a line." Continuing to focus on the point's relation to lines, Dee describes a line as "the race or course of a *Point*," and later stresses that "though it be produced of a Point moued, yet, it doth not consist of pointes" (Billingsley 1570 Sig.*a*2^{r}). In contrast with Recorde, for whom lines are obviously made of points because a dotted line, given enough dots, is observably and intuitively no different from a line drawn with the stroke of a pen, for Dee the converse is equally clear because lines and points, which differ in dimension, divisibility, and magnitude, are of two fundamentally different kinds, and it would be nonsensical to suppose one is made of the other.

As a peculiar yet fundamental philosophical misfit, Dee's point returns in an effusive comparison to its cousin among the numbers, the indivisible unit. Both, he allows, "doo seeme to be much like Originall causes: But the diuersitie neuerthelesse, is great." Points, here, "may haue a certaine determined Situation" whereas "our Vnit is free, and can abyde no bondage." The distinction between points and units helps Dee expound upon the majesty of geometry. For while one can easily see numbers as "a certaine multitude of Vnits" and may likewise begin to imagine what points are with the help of "visible formes," the infinite divisibility of magnitudes is so great as to defy intuition. Even the smallest magnitudes (e.g. "a Line of an inch long"), Dee stresses, can be divided as much as the greatest (e.g. "the diameter of the whole world, from East to West: or any way extended"), and no amount of division will produce a point.

Billingsley, meanwhile, provides the closest sixteenth-century counterpart to Heath's twentieth-century definition with which we began:

---

spanning much of the sixteenth century, mostly from Paris and Basel publishers but including many from

A signe or point is that, which hath no part. (Billingsley 1570, Fol.1^r)

That sentence appears in a larger typeface than the explanatory text that follows it, and is accompanied by a simple diagram in the margin depicting a dot next to a letter "A" that is typographically larger than letters used in other diagrams—perhaps to emphasize the point's smallness. The typographic differentiation between the formal definition and the explanatory text likewise makes the explanation smaller in narrative and authorial terms. Billingsley presents the formal definition as though from the hand of Euclid, making his own elaborations secondary and helping the primary definition to appear stable and unchanging.

And yet, here as in the other texts, this putatively secondary explanation would have been indispensable for vernacular readers to be able to interpret the formal definition. Billingsley begins by explaining that to have no part is to be indivisible. Like Dee, Billingsley elaborates that although the point is not divisible—which is here the defining feature of "quantity," rather than "magnitude"—the point nevertheless pertains to quantity. In this sense, it "is the least thing that by minde and vnderstanding can be imagined and conceyued: then which, there can be nothing lesse, as the point *A* in the margent." In one further paragraph, Billingsley attributes to Pythagoras an interpretation of points in terms of unicity and position and, like Dee, contrasts points and (positionless) numerical units to distinguish magnitudes from numbers.

For all the similarities between Billingsley's interpretation of the point and Dee's, it remains significant that their definitions differed as much as they did.[10] Like Recorde, Billingsley offers a word coupling within his formal definition, joining a literal translation of the

---

further afield.

[10] While Dee and his library were likely among Billingsley's principal sources for his translation, Billingsley worked for the most part from his personally annotated Greek and Latin edition of the *Elements* from Hervagius (1558), now held at Princeton University and showing no annotations that appear to bear on the point's definition.

Greek term (“signe”) to its most dominant counterpart from the Latin tradition (“point”). Not bound to an expectation of matching a Greek or Latin original word-for-word, by contrast, Dee was free to import what he considered the most pertinent properties and interpretations directly into his formal definition. Billingsley’s focus on translation, contrasting with Dee’s philosophical explication, is evident as well in his illustration for the point and its explanation. Where Dee omits an illustration and seems to place the point beyond what can be directly intuited by an individual’s mind, Billingsley offers his readers a tangible image and places the point within his reader’s supposed imaginiation, albeit at one extreme of what can be conceived.

Points in Practice

Works like Recorde’s and Billingsley’s were joined in vernacularizing the principles of Euclid’s *Elements* by a thriving popular scientific literature varying in scope from broad introductions to cosmography to detailed treatises on the techniques and instruments of surveying or navigation. Such works were supported largely by the same publishers who produced weighty tomes like Billingsley’s. In fact, Billingsley’s own publisher repurposed the frontispiece for Billingsley’s *Elements* from one such work from a decade prior, William Cuningham’s 1559 *Cosmographical Glasse*. While Cuningham’s work is scarcely Euclidean in character, he shares Recorde’s preference for the term “prick” in referring to the everyday points of such activities as compass mensuration. In a dialogue explaining the principles of map-making, Cuningham attributes to thirteenth-century Latin commentator Campanus the definition that a point “is that which haue no partes,” and which when moved produces a line (Cuningham 1559, Fol.55). A line, in turn, produces a planar figure when moved, which itself produces a solid body. Rather than bring the point into being by starting with solid or planar figures and working downward in dimension, Cuningham starts with a long-dead authority to narrate his point into being, from which he derives the fundamental features of dimension that justify his account of maps and cosmography.

Leonard Digges was, like Recorde and Cuningham, an ardent exponent of practical geometry. Though his widely-read 1556 *Tectonicon* did not take up the *Elements* explicitly, his posthumously published 1571 *Pantometria*, by contrast, opens with a series of "Definitions, or Elements of Geometry" clearly meant to evoke Euclid before the text veers into its main subject matter of practical and instrumental geometry. The work was finished by Leonard's son Thomas, a leading mathematician and proponent of English vernacular publishing, whose own mathematical education had fallen to Leonard's close acquaintance John Dee after Leonard's death.[11] Thomas appended to Leonard's work his own densely theoretical treatise on geometrical solids, and (although it is unlikely) one cannot rule out Thomas having likewise added the section of definitions, which differ somewhat in style from the ensuing text. The *Pantometria* defines points with a blunt unelaborated statement:

> A *Poynt* I call whiche cannot be diuided, whose parte is nothing. (Digges 1571, Sig.*B*1[r])

The cryptic expression "whose parte is nothing," here, is likely an overly literal transposition of the canonical Greek formulation of the definition. Digges curtly clarifies that this means the point cannot be divided, but offers no further guidance on the matter. Because Digges's geometry does not treat divisibility as such, it is neither necessary nor (I would suggest) intended that the reader understand this definition.

[11] On both Digges's commitments to vernacularization, see Barany (2010, 129).

Rather, Digges's formal definition seems to be more show than substance. His words give the *Pantometria* a Euclidean veneer that reinforces its position of authority, but the real substance of Digges's definition is in its accompanying figure. Adapting an opening definitional figure used in several editions by Basel printer Johann Herwagen, Digges depicts a point, right line, and crooked lines in a series (figure 1; cf. e.g. Hervagius 1558). This scheme can be read without reference to the definitions that surround it, and it imparts nearly everything a reader of Digges's text would need to know about points: that they are small, can come with labels, and are related to lines.

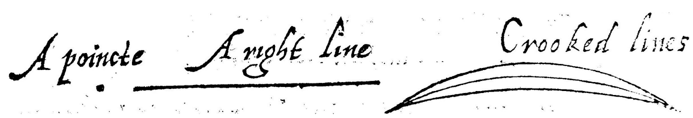

Figure 1: Leonard and Thomas Digges's figure for the point and right and crooked lines in *Pantometria*.

A number of later vernacular practical works paraphrased directly from their more formal predecessors. When Richard Eden (1589) "Englished out of Spanishe" a work on navigation from across the Channel, he liberally borrowed Recorde's coupling of a "poynt or pricke." Billingsley's definition and explanation of the point as the least thing that can be imagined appears at the start of the geometrical outwork to engraver and surveyor Aaron Rathborne's book on surveying, complete with a reference to "this *Point* A" that is nowhere depicted on Rathborne's page (Rathborne 1616, 1; see Taylor 1954, 191). The formulation proved effective for gunner John Babington, whose 1635 *Short Treatise of Geometrie* called the point "that which cannot be divided, and is the least imaginary thing," and illustrated it with an unlabeled typographic period separated from the end of the sentence by a small gap (Babington 1635, 1; see Taylor 1954, 218). William Leybourn, himself a prolific publisher of geometrical works by others, offers a more complete paraphrase of Billingsley's passage at the outset of his own 1653 book on surveying, complete with a figure with a letter "A" above an exaggerated dot, the word coupling "*Pointe* or *Signe*" (with Recorde's "*Point* or *Prick*" later in the paragraph), and a word-for-word transcription of the point's characterization as "the least thing that by minde and understanding can be imagined and conceived, then which there can be nothing lesse, as the *Point*" depicted (Leybourn 1653, 3; see Taylor 1954, 230-231).

Among Leybourn's geometrical authors was surveyor Sylvanus Morgan, whose 1652 *Horlogiographia Optica* takes a rather more effusive approach to the point. Geometry, one learns, "proceeds from the least imaginable thing." Thus,

> *A* Point *is an indivisible*, yet is the first of all dimension; it is the Philosophers *Atome*, such a Nothing, as that it is the very *Energie* of all things, In God it carryeth its extreams from eternity to eternity: in the World it is the same which *Moses* calls the beginning, and is his *Genesis* (Morgan 1652, 2; see Taylor 1954, 223)

So begins a chain of associations expounding on the point as "the birth of any thing" and "our principall significator" at the origin of magnitude and quantity. Morgan's enthusiasm for the point is exceptional in this period, but it is of a piece with the point's well-established narrative pride of place in works on practical geometry, and especially on optics. Morgan's contemporary Robert Hooke turned that role to dramatic effect in his 1665 *Micrographia*, which opens with a microscopic depiction of a speck of dust and the tip of a needle in order to demonstrate his new instrument's powers (see Barany 2012, 157; Schäffner 2005, 70-72).

A Propaedeutic Point

A parallel collection of pedagogical texts built on Billingsley's definitive edition in other ways. Two English authors published major translations of the geometries of noted continental anti-scholastic philosopher Petrus Ramus. A 1590 translation by Trinity College (Cambridge) fellow Thomas Hood rendered the point rather cryptically as "an vndiuisible signe in a magnitude," without elaborating further (Hood 1590, 1; see Taylor 1954, 179). Tottenham parson William Bedwell, on the other hand, "much enlarged" his 1636 translation of Ramus, joining the same definition, numbered and typographically differentiated, to a full paragraph of elaboration. Without quite explaining Ramus's conception of the point as a "signe," Bedwell explains that the point "is not naturall and to bee perceived by sense; Because sense onely perceiveth that which is a body." Most important, here, is that the point lacks magnitude (and so cannot be a body), but serves as "the beginning of all magnitudes"(Bedwell 1636, 9; see Taylor 1954, 194).

London publishers Robert and William Leybourn, the latter of whose 1653 book drew heavily on Billingsley for its definition of the point, produced at mid-century several pedagogical books that were slightly more ambitious in presenting Euclid's first object. The first, from 1651, was Thomas Rudd's translation of the first six books of the *Elements*, to which the publisher added Dee's famous mathematical preface. Its typographically prominent primary definition is very nearly Billingsley's—"*A Point* is that which hath no part"—and its illustration of the point consists of a letter "A" above a very large dot. Indeed, the visual similarities between Rudd's text and Leybourn's later edition leave no doubt that the former provided the template for the latter, and that both were composed with reference to Billingsley's text. But Rudd's explanation of the point departs from Billingsley's, turning instead on an explanation of the Greek term and its literal translation as "*Signe*." For Rudd, the point is a sign because, lacking magnitude, it is merely "a *Signe* of that which is conceived in the minde: It is the same with *Unity* in *Number*, with *Instant* in *Time* and with *Sound* in *Musick*" (Rudd 1651, 3).

This elaboration follows Dee's and Billingsley's identifications of the point with a unit in number, but goes farther than both in calling them one and the same thing. The addition of an instant in time and a sound in music to the litany of fundamental signs, while not originating in the English vernacular corpus, is evidence here of the point's ever-broadening philosophical register in English vernacular works by mid-century, consonant with Morgan's expansive chain of analogies discussed above. As the algebraic geometry most associated with Descartes from a decade and a half prior continued to sweep across Western Europe, the notion that a geometric point and an instant in time would be more or less the same thing was becoming not just a playful conceptual analogy but a powerful principle for the analysis of mechanical phenomena. Rudd's definition would be adopted nearly verbatim in yet another Leybourn edition of the *Elements* authored a decade later by mathematical educators and instrument-makers John Leeke and George Serle. Perhaps having baulked at Rudd's willingness to conflate his variety of exemplary monads, however, Leeke and Serle qualify parenthetically that the point is the same as the others only "in a manner" (Leeke and Serle 1661, 2; see Taylor 1954, 236, 243).

Between Rudd's and Leeke and Serle's editions, Leybourn published two separate geometries by John Newton, an opinionated Oxford-educated proponent of vernacular instruction in elementary mathematics. Both works take lines, not points, to be the fundamental objects of geometry. Newton's 1654 *Institutio Mathematica* scarcely considers the point at all, embedding its definition in an explanation that "The ends or limits of a line are points." It is by virtue of being the end of a line that "a *Point* hath neither part nor quantity"—ends are not parts and do not contain quantities. Here, the line serves implicitly as Rudd's basic kind of quantity, and the point's usual interpretation as the (philosophical) beginning of quantity becomes simply the observation that, as an endpoint, the point appears at the beginning of a line (Newton 1954, 2-3; see Taylor 1954, 225).

Newton's 1660 *Mathematical Elements* makes this principle still more explicit by defining magnitude itself as "either a Line, or something made of a line, or lines." After defining magnitude thusly, Newton offers a definition of a line that includes points as the limits of lines: "For every line is made, continued, and bounded, with a point." While the line's primacy is clearly proclaimed in this work, Newton also gives the point a more dignified standing in its own definition, following the line's. Here, it is "the begining of all continued quantities" and its "imaginary motion" is what produces lines by tracing a course between the line's two ends (Newton 1660, 2-3).

In the same year John Newton published his *Elements*, Isaac Barrow, who is often associated with a somewhat more famous Newton from this period, issued an influential English translation of his own Latin edition of the *Elements*. His compact textbook made extensive use of symbolic short-hands for common terms, making it an efficient reference work and a significant marker of geometry's increasing symbolic reformulation at that time. His definition of the point is simple enough: "A Point is that which hath no part" (Barrow 1660, 1). Indeed, its simplicity is a mark of his era's increasingly stabilised geometric vernacular. For while "pricks" and "signs" remained in circulation when referring to points, there could be little doubt that "point" was the predominant rendering of the term and that narrative devices like word coupling had become superfluous. Barrow's edition is also peculiar in its illustration of the point: a typographic period enclosed in parentheses in the margin next to the definition. His preference for economical expression may have ruled out labeling his point, but an isolated dot on the page may have looked too accidental.

Points of Contention

Despite its plurality of interpretation, the point's definition did not, for the most part, become a point of open contention for early modern English geometers. If anyone were to pick a fight over that which has no part, however, it would be the famously belligerent Thomas Hobbes.[12] In a 1656 English translation of his *Elements of Philosophy*, first published in Latin in 1655 (and better known by its Latin subtitle: *De Corpore*), Hobbes defines the point in the eighth chapter, on "*Body* and *Accident*." Hobbes's chief interest here is in bodies in motion, from which points and lines arise as mathematical simplifications. Thus, if a body's magnitude "be not at all considered" then its course is a line and "the Body it self, a POINT; in which sense the Earth is called a *Point*" (Hobbes 1656, 81).

The distinction between the abstractions of one's imagination and the phenomena of nature proves important three pages later, where Hobbes defines the difference between bodies and accidents. Here, "we may feign in our Mind that a Point may swell to a huge bulk, and that this may again contract it selfe to a Point," but such mental gymnastics turning nothing into something and back again into nothing cannot "possibly be done in Nature" (84). Thus, bodies can be neither created nor destroyed, whereas accidents are ephemeral things that can be. Hobbes returns briefly again to the point in his twelfth chapter, on "Quantity." There, he observes that by recording the motion of an object ("graven in some durable Matter") or by applying objects of the same dimension to each other ("by *Apposition*"), one can pass between objects of various dimensions. One can thus "describe a Line by Points" with points here "to be understood [as] very short Lines" (103). This interpretation is reinforced by a later consideration of the limits of a line's divisibility. Here, though one cannot find a "least straight line," one can nevertheless produce a straight line "so small, as is not at all considerable; which we call a Point" (136).

In three passages, then, Hobbes offers three related conceptualisations of the point: as a mathematical simplification of position; as a "Nothing" that can only be related to bodies in the imagination, and not in nature; and as a very short line, in the sense that it can produce lines through motion or apposition, or that one can divide a line to produce a segment of negligible length. Hobbes appended to his 1656 translation a response in English to a number of criticisms the original Latin work had received from authors also writing in Latin, principally John Wallis. The centerpiece of his response was a point-by-point rehearsal of the first definitions of Euclidean geometry, beginning of course with the point.

Hobbes takes as his opponents' view the stipulation that the point has no quantity, which he caricatures with the extreme formulation that "a Point is nothing." In response, he launches a systematic consideration of the point's Latin definition, which he initially renders into English as "*a Marke is that of which there is no part*" (Hobbes 1556, 'Six Lessons,' 5). One cannot leave things at that, Hobbes claims, for

> Theologers say the Soul hath no part, and that an Angel hath no part, yet do not think that Soul or Angel is a point. A mark, or as some put instead of it [*s‹gmh*], which is a mark with a hot Iron, is visible; if visible then it hath Quantity, and consequently may be divided into parts innumerable. That which is indivisible is no Quantity; and if a point be not Quantity, seeing it is neither substance nor Quality, it is nothing.

Rather, Hobbes insists, a point is not an indivisible nothing but a genuine quantity that remains undivided; that is, "*a Body whose Quantity is not considered.*" Thus, an object is a point not for what it intrinsically contains but for how it is used in a demonstration. Hobbes returns to

[12] The point represented just one dimension of Hobbes's bitter dispute with John Wallis; see Jesseph (1999).

this claim in order to comprehend Euclid's other stipulations about points, which to him make sense only where a point is a small body whose quantity, though not nothing, is not considered ('Six Lessons,' 6, 9). Where Dee, for instance, revels in the infinite exactitude of geometry that fundamentally separates the point from the line, Hobbes insists on a radically different interpretation (a philosophically dogmatic version of Recorde's) where the point is little more than an arbitrary terminus to one's reckoning about sensible things.

Wallis wasted no time in responding, this time in vernacular, to Hobbes's indictment. To start, he expresses bafflement that Hobbes "railes upon me, twenty times over, as if I had somewhere said *A point is nothing*" and professes only to have given Euclid's literal definition, which he reproduces untranslated in Greek (Wallis 1656, Sig.*A*4$^{\mathrm{v}}$). He continues this linguistic evasion with a criticism of Hobbes's Latin grammar, claiming that Hobbes has incorrectly rendered his own interpretation that the point is a body whose quantity is neglected (11-12). Moreover, Wallis points out, Hobbes seems to contradict his assertion that the point must be a body when he claims that it "is neither substance nor Quality."

Wallis returns to the theme of mistranslation in an extended discussion ridiculing Hobbes's depiction of a point as a mark with a hot iron. First, he suggests the argument rests on no more than a pun: that something that is "*visible,* therefore 'tis *divisible*" (28). The term [*Stigm'h*], writes Wallis, "signifies a *distinctive point* in writing, made with *pen* or *quill*, not a mark made with a *hot Iron*" and also denotes "a Mathematicall Point; or somewhat else that is very small" such as a moment in time. Wallis then suggests that a schoolboy's lashing might improve Hobbes's command of Greek grammar and vocabulary (29).

The most direct response to Hobbes's formulation of the point, however, comes in response to his passage on the definition of a terminus. Where Hobbes insists that a terminus must itself be a body, Wallis asserts that if a terminus could be divided then its parts “cannot be all utmost; but one must stand beyond another” (30). Wallis amplifies this objection with a simple rejoinder to Hobbes’s initial definition of a point as the location of a moving body—“But what if a Body be not moved?”—and later quotes Hobbes contradictorily considering “the Point which is not moved” (52, 79).

Points were sufficiently central, at least rhetorically, to the dispute between Hobbes and Wallis that Hobbes’s next rejoinder took the term “Markes,” i.e. points, for its title.[13] Here, Hobbes marshals Aristophanes to his interpretation of the point as the mark of a hot iron and reiterates that a point is a visible mark (Hobbes 1657, 15). As for Wallis’s insistence on marks in pen or quill as the better translation, Hobbes excerpts from an anonymous letter that asserts that pens and quills did not date to antiquity, and so could not have underwritten Wallis’s interpretation of the Greek term in question (28). The letter unsurprisingly goes on to clarify that the point is a visible “prick or marke” made “with any thing in any manner,” and it refers to several Classical references to hot iron branding (28-29).

The back-and-forth between these two fiery interlocutors underscores the potential depth of disagreement largely unrealized in the first English renderings of the Euclidean point from a century prior. While Recorde and Dee may well have stopped short of the extreme positions advocated by, respectively, Hobbes and Wallis, the essential points of disagreement among the latter are very much to be found in the geometries of the former. Moreover, the debate underscores just how flexible and open-ended the lexicographic and mathematical bases for interpretations of Euclidean geometry could be. Whether one or the other had a more correct reading of Classical sources or contemporary geometry, both confidently asserted that their

[13] The full title is *Markes Of the Absurd Geometry, Rural Language, Scottish Church-Politicks, And*

respective readings were amply supported and the only ones that were justifiable. In no sense could one say the point had a straightforward and inevitable stock of pre-given interpretations simply awaiting vernacularization.

Conclusion

To be sure, amidst all the divergent renderings of a supposedly unitary text in the first century of English vernacular geometry, there were many points in common. Though they disagreed about how small it was, all agreed the point was extremely small. Most allowed that the point was indivisible, although they had different ideas about what this meant. Few would protest the point's conventional depiction as a dot. The foregoing has not been to argue that the Euclidean point was an anything-goes proposition in the open frontier of a new language.

Quite to the contrary, Euclid's English translators built on each others' works, and on works in diverse foreign languages, new and old. Where they disagreed, they could for the most part parse it as a matter of taste and presentation rather than as a fundamental difference of interpretation. There is no evidence that the lack of a single unified English counterpart to the by-then canonical expressions in Latin or Greek weighed heavily on any of the above-considered authors.

All this serves to make the point's pervasive interpretive disunity all the more provocative. Rather than persist as an absolute and syllogistic foundation for Euclidean geometry, the point existed in English vernacular geometry as a loosely-integrated cloud of consensual interpretations and implications. That it was rarely a matter of open dispute is, here, a

Barbarismes of John Wallis. The pun is clearly intended in Hobbes's text.

mark not of its firm grounding in axiomatic truth but of its pliability in its great variety of rhetorical settings.

And yet, with the possible exception of Recorde, nobody seemed to see the point this way. Each different definition appeared in its own context as *the* definition of a point. When these definitions coexisted peaceably, all was well. When, in the case of Hobbes and Wallis, they became points of vociferous dispute, the one thing that was not disputed was that the point was, in the end, just one thing. When the point's vernacular formulation changed over time, the one thing that did not change was the point's formulation as something timeless.

Three narrative features, I claim, made this paradoxical circumstance possible. First, the formal stability of the point's formal definition—first in Greek and Latin, and in English by the end of the point's first century in English—made fundamental interpretive disagreements appear instead as secondary elaborations on a shared idea. As a single idea could have multiple elaborations, this made the point appear more consensual than I suggest was the case in theory and practice.

Second, analogous to the point's lexical stability, the visual stability of the point as a dot in print or manuscript united authors in a single custom of depiction, even while their notions of what was being depicted could differ. Printers displayed dots in many different sizes and placements, but points were always recognisably dots. While publishers in later centuries would omit dot illustrations of the Euclidean point in their geometric books, dots offered a dominant and unifying visual trope for the first century of English vernacular geometry.

Third, the point's very narrative flexibility may well have helped its expositors sustain the idea of the point's unicity by deferring and deflecting such challenges as might arise. Practical and pedagogical genres, while often supporting fine-grained parsing for key ideas, are rarely explicitly adversarial. Authors took what they needed of the point as a solid-enough foundation for their particular projects. When such foundations were called into question—as with Hobbes and Wallis—their solid-enough constitution could prove strikingly fragile.

Historians of science have long recognised the contingency and consensual constitution of facts and of generalisations about the natural world, but they have been less ready to turn their attentions to the ideal world of syllogistic knowledge. But even our most basic ideas—the ones we consider foundational and which we place at the heart of great monuments of philosophical theory and practice—must be recognised as contingent collective productions. Precisely because of their privileged discursive place, such fundamental notions rarely brook the kind of variation and dispute so useful to the historiography of contingent things. The Euclidean point, the beginning of magnitude, was one such fundamental anchor for knowledge and agreement. Its history is a reminder that such consensus is never inevitable, is always the result of points borrowed, elided, and deferred.

Sources

Babington, John. 1635. *A Short Treatise of Geometrie.* London: Thomas Harper..

Barany, Michael J. 2010. "Translating Euclid's Diagrams into English, 1551–1571." In *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics,* edited by Albrecht Heeffer and Maarten Van Dyck, 125-163. London: College Publications.

Barany, Michael J. 2012. " 'That small and unsensible shape': Visual Representations of the Euclidean Point in Sixteenth-Century Print." *Spontaneous Generations* 6:148–159.

Barrow, Isaac. 1660. *Euclide's elements the whole fifteen books compendiously demonstrated*. London: R. Daniel.

Barrow-Green, June. 2006. " 'Much necessary for all sortes of men': 450 years of Euclid's Elements in English." *BSHM Bulletin* 21(1):2–25.

Bedwell, William. 1636. *The Way to Geometry.* London: Thomas Cotes.

Billingsley, Henry. 1570. *The Elements of Geometrie.* London: John Daye.

Cuningham, William. 1559. *The Cosmographical Glasse.* London: John Daye.

Digges, Leonard. 1571. *A Geometrical Practise, named Pantometria.* Edited by Thomas Digges. London: Henrie Bynneman.

Eden, Richard. 1589. *The Arte of Nauigation,* from a text by Martin Cortes. London: Abell Jeffes.

Heath, Thomas L. 1956. *The Thirteen Books of Euclid's Elements*, 2nd edn, vol 1. New York: Dover.

Hervagius, Johann. 1558. *Euclidis Megarensis mathematici clarissimi Elementorum geometricorum libri XV.* Basel.

Hobbes, Thomas. 1656. *Elements of Philosophy.* London: R. & W. Leybourn.

Hobbes, Thomas. 1657. *Markes Of the Absurd Geometry, Rural Language, Scottish Church-Politicks, And Barbarismes of John Wallis Professor of Geometry and Doctor of Divinity.* London.

Hood, Thomas. 1590. *Elementes of Geometrie.* London: John Windet.

Høyrup, Jens. 1996. "The formation of a myth: Greek mathematics—our mathematics." In *L'Europe mathématique / Mathematical Europe,* edited by Goldstein, Gray, and Ritter, 103-122. Paris: Éditions de la Maison des sciences de l'homme.

Jesseph, Douglas M. 1999. *Squaring the Circle: The War between Hobbes and Wallis.* Chicago: University of Chicago Press.

Johnson, Francis R. 1944. "Latin versus English: The Sixteenth-Century Debate over Scientific Terminology." *Studies in Philology* 41(2):109–135.

Johnson, Francis R. and Sanford V. Larkey. 1935. "Robert Recorde's Mathematical Teaching and the Anti-Aristotelian Movement." *The Huntington Library Quarterly* 7:59–87.

Leeke, John and George Serle. 1661. *Euclid's Elements of Geometry In xv. Books*. London: R. & W. Leybourn.

Leybourn, William. 1653. *The Compleat Surveyor.* London: R. & W. Leybourn.

Mancosu, Paulo. 1996. *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century.* Oxford: Oxford University Press.

Morgan, Sylvanus. 1652. *Horlogiographia Optica.* London: R. & W. Leybourn.

Netz, Reviel. 1998. "Deuteronomic Texts: Late Antiquity and the History of Mathematics." *Revue d'histoire des mathématiques* 4:261–288.

Newton, John. 1654. *Institutio Mathematica.* London: R. & W. Leybourn.

Newton, John. 1660. *Mathematical Elements in III Parts.* London: R. & W. Leybourn.

Rampling, Jennifer M. 2011. "The Elizabethan Mathematics of Everything: John Dee's 'Mathematicall Praeface' to Euclid's Elements." *BSHM Bulletin* 26(3):135–146.

Rathborne, Aaron. 1616. *The Surueyor in Foure Bookes.* London: W. Stansby.

Recorde, Robert. 1551. *The Pathway to Knowledg.* London: Reynold Wolfe.

Richards, Joan L. 1988. *Mathematical Visions: the pursuit of geometry in Victorian England.* Boston: Academic Press.

Roberts, Julian and Andrew G. Watson (eds). 1990. *John Dee's Library Catalogue.* London: Bibliographical Society.

Rommevaux, S., Djebbar, A. and Vitrac, B. 2001. "Remarques sur l'Histoire du Texte des Éléments d'Euclide. *Archive for History of Exact Sciences* 55(3):221-295.

Rommevaux, Sabine. 2009. “The Transmission of the Elements to the Latin West: Three Case Studies.” In *The Oxford handbook of The History of Mathematics,* edited by Eleanor Robson and Jacqueline Stedall, 687-706. Oxford: Oxford University Press.

Rudd, Thomas. 1651. *Euclides Elements of geometry the first VI books.* London: R. & W. Leybourn.

Schäffner, Wolfgang. 2005. “The Point: The Smallest Venue of Knowledge in the 17th Century (1585–1665).” In *Collection, Laboratory, Theater: Scenes of Knowledge in the 17th Century,* edited by Helmar Schramm, Ludger Schwarte, and Jan Lazardzig, 57-74. Berlin: Walter de Gruyter.

Taylor, E.G.R. 1954. *The Mathematical Practitioners of Tudor & Stuart England.* Cambridge: Cambridge Unviersity Press.

Vitrac, Bernard. 2012. “The Euclidean Ideal of Proof in *The Elements* and Philological Uncertainties of Heiberg’s Edition of the Text.” In *The History of Mathematical Proof in Ancient Traditions,* edited by Karine Chemla, 69-134. Cambridge: Cambridge University Press.

Wallis, John. 1656. *Due Correction for Mr Hobbes.* Oxford: Leonard Lichfield.