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Harris, Michael [Harris, Michael Howard] (F-PARIS7)
★**Mathematics without apologies.**
Portrait of a problematic vocation.
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Mathematics without apologies is a wide-ranging and significant work of cultural criticism. Drawn from Harris's personal and professional experiences as a mathematician and his omnivorous and often sophisticated amateur engagement with philosophy, literary theory, sociology, history, popular culture, and much else, the book is fascinating and frequently frustrating. Harris aims to provoke, raising important and timely questions about the discipline of mathematics, the nature of its idiosyncratic enterprise, and its place in modern societies and economies. While his answers typically fall short of his provocations, Harris nonetheless conjures a fruitful array of suggestive and challenging interventions that richly reward the reader who brings the background, curiosity, and patience to follow them.

Harris's title self-consciously evokes G. H. Hardy's classic *Mathematician's Apology* [Cambridge Univ. Press, Cambridge, England, 1940; MR0003386 (2,210g)]. Invited to review Hardy's book for the recently established Mathematical Reviews, George Pólya instead selected a passage from its fifth page. In that quote, Hardy announced his intention to ask "What is the proper justification of a mathematician's life?" and added that "my apology is bound to be to some extent egotistical". Hardy's question and qualification alike apply to Harris's book, to which one should add Harris's explanation that "it is primarily an account of mathematics as a way of life" (p. 6). Though these quotes state the authors' aims accurately enough, in both cases they largely miss what makes the works distinctive and valuable.

Here, Harris's subtitle is a better indication of his quarry. As a "portrait", the book embraces its author's limited perspective, with all its biases and blind spots, in order to depict selected features of a mathematical life in greater richness and complexity, without the burden of aspiring to comprehensiveness or representativeness. That Harris is a decidedly unusual mathematician, particularly with regard to the combination of critical appetites and faculties on display in this book, both adds to the book's coherence and brings to light many notable aspects of mathematics and mathematicians that would be lost in a more balanced treatment. With "vocation", a freighted technical term borrowed from sociologist Max Weber, Harris signals his interest in how mathematics operates simultaneously on and through individuals, communities, and societies in ways that blend ethical, cultural, intellectual, and practical norms and values.

Mathematics is a "problematic" vocation in its literal orientation around posing and solving mathematical problems, but also in the many moral, social, economic, and other problems those who use and study mathematics tend to encounter, distort, or elide. Marshaling such sources as Goethe's *Faust*, Grothendieck's *Récoltes et Sémailles*, and Nietzsche's reflections on spirituality and uncertainty, Harris stresses how the problems and problematicity of mathematics are intertwined in its culture, narratives, ethics, history, material conditions, and social positions. Harris's unstinting insistence on the mathematical problematic, in its manifest complexity, may be the book's greatest strength.

The book consists of three parts, divided into ten chapters of widely varying length, among which Harris intersperses a series of "sessions" under the rubric of "How to

Explain Number Theory at a Dinner Party". Harris starts with a long and thoughtful preface that explains his scholarly posture with uncommon candor, followed by a short introduction. The first substantive chapter introduces what Harris calls "routinized charisma", which for him is the social glue that holds the mathematical research community together and which structures the vocational pursuits of its participants. His next chapter skeptically surveys the background and assumptions behind justifications for mathematics found in apologies like Hardy's, namely that the subject is "good, true, and beautiful". Harris then considers mathematicians' ethical and technical roles in quantitative finance before turning, in a short "bonus chapter", to a playful interpretation of Thomas Pynchon's novels that demonstrates a number of metaphoric uses of mathematics, some of which reappear later in other guises.

Harris begins the book's second part by exploring the historical and popular connections between mathematics, love, and sex. There follows a probing chapter on the shifting and disputed status of mathematical foundations, dramatically depicting category theory and the Langlands program in relation to some areas of Eastern philosophy and metaphysics. A chapter on "tricks" in mathematics then examines attitudes toward mathematical proofs and techniques before turning to mathematics in elite and popular genres of music.

These middle chapters, in their extraordinary topical, conceptual, and methodological breadth, exemplify Harris's distinctive approach. Harris is emphatic that non-mathematicians have had important and insightful things to say about the discipline, but also that they have been limited to a significant degree by the kinds of public representations mathematicians have made about themselves. His expositions thus mingle an eclectic mix of ideas and source materials from a wide range of traditions, juxtaposing them largely on the basis of their thematic suggestiveness in a way that can sometimes seem unmotivated. The results are frequently striking, but often depend on the reader's ability to place references in context. This difficulty is perhaps the inevitable byproduct of Harris's commitment to so expansive a treatment of mathematics, which is especially notable in his engagement with contemporary sources and others well beyond the usual "about mathematics" canon. Harris is necessarily a non-expert on this vast trove, and his is certainly not a specialist's account, nor does it aim to be. Conversely, because Harris draws from so many scholarly and cultural idioms, nearly every reader will find parts that are inaccessibly obscure, although in many instances these have the happy effect of promoting a desire to learn more about unfamiliar matters. Indeed, confronting and working around obscurities seems very much a part of the mathematical life Harris aims to convey, as is the pleasure of finding insight in unexpected places.

In the third part, a short chapter about a career-shaping dream is awash in technical allusions and neatly bookends Harris's early chapter on charisma. Harris concludes with a renewed consideration of the present justifications for the discipline and profession of mathematics. He returns, in particular, to ethologist Gordon Burghardt's notion of play in a "relaxed field", discussed principally in chapter 3 (on the "good, true, and beautiful"), which comes to represent the elusive value of the pleasure found in doing mathematics without restraint or apology. An afterword encapsulates the book's approach and themes in the life and thought of topologist Felix Hausdorff, who offers Harris an evocative alternative to the vision of mathematics associated with Hausdorff's contemporary David Hilbert.

The dinner party "sessions" are Harris's attempt to explain in lay terms what his mathematical research involves and what makes it interesting. In the sessions he covers prime and transcendental numbers, solving equations, analyzing equations through congruences, and some elements of the theory of elliptic curves. His basic structure, which he does not quite maintain throughout the book, pairs a brief mathematical exegesis

with a dialogue between two imagined interlocutors at a dinner party, Performing Artist and Number Theorist, whose challenges and responses elaborate key ideas and themes from the exegesis. While these parts of the book seem unlikely to help the reader explain number theory at a future dinner party (unless it is that rare dinner party where the philosophy of Ludwig Wittgenstein counts as a natural route to defining rational numbers), they do raise a number of questions and analogies that may enrich one's own understanding of what number theorists do and why they do it.

For all Harris packs into *Mathematics without apologies*, the book feels at the same time very unfinished. Some of this impression is the fault of Princeton University Press, which brought the book out with a distracting number of copyediting errors and inconsistencies. Though Harris urges that "the reader should not mistake this book for a work of scholarship" (p. xviii), his unevenly executed scholarly apparatus of notes and citations occupies nearly the last quarter of the volume, and would have benefited from greater editorial discipline. Some of the impression of incompleteness owes to Harris's style and approach, which avowedly favors provocations over syntheses. Interested readers would do well to follow the blog Harris has maintained while promoting the book, which he uses to elaborate a number of topics and themes omitted or underdeveloped in the published text, and which itself also exemplifies many features of the mathematical life Harris discusses therein.

Playful, erudite, probing, and difficult, *Mathematics without apologies* is not an ordinary contribution to any of the familiar genres of writing about mathematics that fill Mathematical Reviews. For better and occasionally for worse, Harris's unusual approach makes mathematics out to be a discipline at once pervasive and elusive, superficially graspable yet impossible to hold in place: in a word, problematic. Yet in the face of mathematics' unending problems Harris sustains a remarkable and heartening personal and ethical conviction that mathematics is worthwhile, important, and needs no apology. If there is one lesson in this book, it is that ethical clarity and problematic difficulty are not in competition, but are two interlocking features of the same pursuit.

Michael J. Barany