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'Chasing' the diagram—the use of visualizations in algebraic reasoning. (English. English summary)

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In this cogent philosophical article, De Toffoli offers a framework for evaluating mathematical notations and then applies it to the specific notational form of the commutative diagram in homological algebra. The article's characterization of mathematical notation in terms of three criteria—expressiveness, calculability, and transparency—provides a fruitful perspective on the hybrid qualities of diagrammatic and symbolic reasoning in mathematics. The three criteria correspond, respectively, to a notation's capacity for semantic representation, its amenability to operations and manipulation, and its relationship to various aspects of training and cognition that condition mathematical inquiry and understanding. Returning repeatedly to these three features, De Toffoli shows their wide applicability to the philosophy of mathematical representation while using them to identify several distinctive characteristics of commutative diagrams. These diagrams, De Toffoli argues, constitute an effective (indeed, in some respects indispensable) notational system that mathematicians navigate dynamically like a map in order to formulate and establish results. The method of 'diagram chasing' shows the hybrid semantic and cognitive character of such diagrams and their uses, which De Toffoli discusses in general terms and also exemplifies through two specific mathematical arguments using commutative diagrams.

De Toffoli persuasively presents these claims as an intervention in the recent body of scholarship identified as the Philosophy of Mathematical Practice [see *The philosophy of mathematical practice*, Oxford Univ. Press, Oxford, 2008; MR2590934]. The article engages extensively with several themes from the recent literature on visual aspects of mathematics and logic, including especially those of M. Giaquinto [*Visual thinking in mathematics*, Oxford Univ. Press, Oxford, 2007; MR2345085], D. Macbeth [Philos. Sci. (Paris) **16** (2012), no. 1, 29–54; MR3026945; Philos. Math. (3) **20** (2012), no. 1, 58–85; MR2889175], and their interlocutors, as well as the author's prior collaborations with V. Giardino [Erkenntnis **79** (2014), no. 4, 829–842; MR3260948; in *From logic to practice*, 315–336, Boston Stud. Philos. Hist. Sci., 308, Springer, Cham, 2015; MR3329904]. The findings here compare suggestively both in general claims and particular observations to a much broader literature on representation in scientific practice [e.g. *Representation in scientific practice*, MIT Press, Cambridge, MA, 1990; *Representation in scientific practice revisited*, MIT Press, Cambridge, MA, 2014; D. I. Kaiser, *Drawing theories apart*, Univ. Chicago Press, Chicago, IL, 2005; MR2361711] which does not appear in the author's discussion. To take one small but provocative example, the author's several passing analogies between commutative diagrams and maps of the London Underground might offer a striking extension of Science Studies scholarship on cognitive and practical dimensions of that very map [see J. Vertesi, Soc. Stud. Sci. **38** (2008), no. 1, 7–33, doi:10.1177/0306312707084153].

Analytically, De Toffoli focuses on classification and evaluation, integrating the article's analysis with relevant touchpoints in the recent Philosophy of Mathematical Practice literature. As in much of that literature, the author's evidence regarding 'practices' themselves appears largely hypothetical or ideotypical. Notwithstanding the handful of well-chosen examples that illustrate the phenomena in question, most assertions about how mathematicians use specific representations must be taken on the author's authority (to the extent the reader does not recognize them from personal experience). De Toffoli soundly justifies an expansive approach to questions about cognition in this article, but this creates its own problems of evidence. Claims about matters such

as externalization, cognitive load, instrumental necessity, and ease or difficulty of interpretation are generally plausible, but often seem *ad hoc* or circular in the expository and programmatic confines of the present work. Nevertheless, De Toffoli has outlined and illustrated a promising approach to understanding such cognitive and related questions about mathematical representation, an approach clearly meriting further investigation.

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