"Some call it Arsmetrike, and some Awgryme"
Misprision and precision in algorithmic thinking and learning in 1543 and beyond.

Michael J. Barany, University of Edinburgh
michael@mbarany.com, http://mbarany.com

Abstract
Algorithms found significant new audiences in early modern mathematical pedagogy through vernacular arithmetics like Robert Recorde's 1543 *Ground of Artes.* Recorde presented rule-based algorithmic methods as powerful but error-prone, requiring practice and expertise to apply effectively. This view of algorithms is apparent in Recorde's exposition through pedagogical dialogue, with a frequently-erring Scholar continually corrected by a learned Master who insists that practice, experience, and situational adaptation were paramount, superseding the "light knowledge" of rules learned only mechanically or by rote. In contrast to later mechanistic understandings of algorithms, sixteenth-century algorithmic thinking reflected expert judgement and discernment, not just in the manipulation of numbers but in the contextual appreciation of mathematical knowledge and its applications and limitations. This essay explores these aspects of early modern algorithmic thinking through a close reading of Recorde's presentation of long division, framed by Recorde's discussion of the terminology of arithmetic and algorism, synonymous in this period with algorithms, and concludes with a meditation on farts and algorithmic modernity.

From Arses to Algorisms
Robert Recorde's 1543 arithmetic primer *The Ground of Artes* taught anglophone readers the art and science of numbers and their innumerable uses, promising in its subtitle an "easyer & exacter" exposition "moch necessary for all states of men."[1] While not the first English vernacular arithmetic, the text and subsequent editions revised and expanded by Recorde himself and by other notable authors stood out at a time of burgeoning vernacular printed science and remained in print for more than a century and a half.[2] Historians have long regarded the work as a significant milestone in the histories of education, numeracy, and arithmetic.[3] Often

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[1] Robert Recorde, *The grou[n]d of artes teachyng the worke and practise of arithmetike, moch necessary for all states of men* (London: R. Wolfe, 1543), STC (2nd ed.) 20797.5, accessed on Early English Books Online [EEBO]; Stephen Johnston, "Recorde, Robert (*c.*1512–1558)," *Oxford Dictionary of National Biography* (Oxford University Press, 2004), doi:10.1093/ref:odnb/23241. When quoting Recorde, I have substituted j for i and v for u where warranted and expanded contractions with bracketed letters, but have otherwise preserved Recorde's spelling. Readers without access to EEBO can follow most of this chapter's discussion of the text with a free digital version of a later edition, e.g. the 1618 edition in Google Books, https://books.google.com/books?id=i8NJomIVzlgC (accessed 2019).
[2] Joy B. Easton, "The Early Editions of Robert Recorde's *Ground of Artes*," *Isis* 58(4), 1967, pp. 515-532; John Denniss and Fenny Smith, "Robert Recorde and his remarkable Arithmetic," in Gareth Roberts and Fanny Smith, ed., *Robert Recorde: the life and times of a Tudor mathematician* (Cardiff: University of Wales Press, 2012), pp. 25-38; John Denniss, "Learning arithmetic: textbooks and their users in England 1500-1900," in *Oxford Handbook of the History of Mathematics*, ed. Eleanor Robson and Jacqueline Stedall (Oxford: Oxford University Press, 2009), pp. 448–467.
[3] Keith Thomas, "Numeracy in Early Modern England," *Transactions of the Royal Historical Society* 37, 1987, pp. 103-132; Francis R. Johnson and Sanford V. Larkey "Robert Recorde's Mathematical Teaching and the Anti-Aristotelian Movement," *The Huntington Library Quarterly* 7, 1935, 59-87; E.G.R. Taylor,

erroneously credited with inventing the "=" sign for equivalence,[4] Recorde incontrovertibly left a lasting mark on pedagogy, terminology, and symbolism through his coinages, explanations, and popularizations. Recent scholarship has situated *The Ground of Artes* (along with Recorde's other works) in the histories of reasoning and computation, as well, with a 2011 Recorde biography even appearing in a History of Computing book series.[5]

The book's Pembrokeshire-born and Oxbridge-educated pun-plying polymath author, one of British mathematics and science's most important early vernacularizers and expositors, chose his words with care and wit.[6] The book's opening dialogue begins with the Master impressing upon the Scholar (with a clearly intended pun) numbers' "unnumerable" "co[m]modities" as "the ground of all me[n]ns affayres"(1^v). Summarizing in verse, "yf nombre be lackynge, it maketh men dumme, so that to most questions, they must answere mum" (2^v). After an extended exchange on the value of numbers, the Master asks the Scholar "what call you the science, that you desyre so greatly" (6^r). Despite having named it unproblematically moments earlier in the dialogue (3^r), and having encountered the term no fewer than eleven times from the Master in the interim (3^r-5^r), the Scholar muffs the quiz: "Some call it Arsmetrike, and some Awgryme" (6^r).

Word choices like these carried significant implications in Recorde's writing, and the author's attention to etymology and flair for coinage allow one to impute meaning even in errant phrases offered so as to be corrected. Variants of both of the Scholar's terms can be found in English manuscripts dating to the fifteenth century and earlier, derived through multiple renderings from Greek (by way of Latin) and Arabic, respectively, and one scholar has suggested a Welsh influence in Recorde's latter term, as well.[7] If they were not necessarily in the air,

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The Mathematical Practitioners of Tudor & Stuart England (Cambridge: Cambridge Unviersity Press, 1954), 15, 167; S. K. Heninger Jr., "Tudor Literature of the Physical Sciences," *The Huntington Library Quarterly* 32(2), 1969, pp. 101-133, on 107-110.

[4] E.g. Florian Cajori, *A History of Mathematical Notations, vol. I: Notations in Elementary Mathematics* (London: Open Court, 1928), p. 164; Jack Williams, *Robert Recorde: Tudor Polymath, Expositor and Practitioner of Computation,* History of Computing (London: Springer Verlag, 2011); Gordon Roberts, *Robert Recorde: Tudor Scholar and Mathematician* (Cardiff: University of Wales Press, 2016); "Equals sign," *Wikipedia,* online, https://en.wikipedia.org/w/index.php?title=Equals_sign&oldid=767759138, accessed 2017; J. J. O'Connor and E. F. Robertson, "Recorde Summary," *MacTutor History of Mathematics,* http://www-history.mcs.st-and.ac.uk/Mathematicians/Recorde.html, accessed 2017. Without being able to say with certainty whether Recorde knew of them, Jens Høyrup identifies Italian manuscript examples predating Recorde's print use by more than half a century. Jens Høyrup, "Hesitating progress—the slow development toward algebraic symbolization in abbacus-and related manuscripts, c. 1300 to c. 1550," in *Philosophical Aspects of Symbolic Reasoning,* ed. Albrecht Heeffer and Maarten Van Dyck (London: College Publications, 2010), pp. 3-56, on 36-37.

[5] Lisa Wilde, "'Whiche elles shuld farre excelle mans mynde': Numerical Reason in Robert Recorde's Ground of Artes (1543)," *Journal of the Northern Renaissance* 6, 2014, online, http://www.northernrenaissance.org/whiche-elles-shuld-farre-excelle-mans-mynde-numerical-reason-in-robert-recordes-ground-of-artes-1543/; Roberts and Smith, ed., *Robert Recorde,* op. cit.; John V. Tucker, "Foreword: Robert Recorde and the History of Computing," in Williams, *Robert Recorde,* op. cit., pp. v-xi.

[6] On Recorde's vernacularization, see Michael J. Barany, "Translating Euclid's Diagrams into English, 1551-1571," in *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics,* ed. Heeffer and Van Dyck, op. cit., pp. 125-163, on 128-137.

[7] "augrim," *The Middle English Dictionary,* online, http://quod.lib.umich.edu/cgi/m/mec/med-idx?type=id&id=MED3019, accessed 2017; "algorism, n." and "arithmetic, n.1," *OED Online* (Oxford

neither were they pulled from nowhere. To those who may have encountered these names in the past, the Master's correction set them right. "Bothe names are corruptly writen," asserts the Master, "Arsmetrike for Arithmetyke (as the Grekes call it) and Awgrym for Algorisme (as Arabyans sounde it) whiche bothe betoken the science of nombrynge" (6^r). Indeed, the Oxford word oracle places Recorde's 1543 text at the culmination of a centuries-long chain that "gradually corrected" *arsmetrike* to the current standard of *arithmetic,* and notes a fifteenth-century arithmetic text that contrasted *algorym* to the "lewder use" of *Augrym*.[8]

Those seeing the Scholar's corruptions for the first time could still recognize something in the words. The first presented a pun: science of numbers, meet the mensuration of arses.[9] This pun had a distinguished pedigree in English letters, figuring pivotally in the *Canterbury Tales,* where Chaucer invokes "ars-metrike" in the *Summoner's Tale* in face of the problem of dividing a fart equally.[10] The second misprision looked strange, and the Master's correction in terms of how it was "written" draws attention to this alien orthography. But to appreciate the full measure of its strangeness one should try to say it. As "the Arabians" say it, algorism is new and foreign. As the Scholar misconstrues it, the term is something of a mumblesome mouthful, a kind of garbled speech well matched in the period's rhetorical traditions to the inarticulate expressions associated with the arse.[11]

At bottom, Recorde propounded arses and mumbles to make a point about pedagogy, a point this chapter shall interpret through Recorde's algorithmic-arithmetic practice in *The Ground of Artes*. Recorde's preface justifies using the "fourme of a dyaloge, bycause I judge that to be the easyest waye of enstructio[n], when the scholer may aske every doubte orderly, and [the] mayster may answere to his questio[n] playnly."[12] This "reason of ryght teachynge" constitutes a claim about how to learn and think. *The Ground of Artes* proceeds dialogically through mistakes and corrections, training readers to figure correctly while also insisting that right thinking comes from practice, discernment, and mastery, not strict uncomprehending adherence to fixed procedures. While related to later evolutionary notions of trial and error, themselves joined by a winding genealogy to early modern arithmetic,[13] Recorde's dialogical didactic form should be understood in the context of customs and assumptions from early

University Press, 2016). Nia M. W. Powell, "The Welsh context of Robert Recorde," in Roberts and Smith, ed., *Robert Recorde,* op. cit., pp. 123-144, on 130-131, 140 (n. 39).

[8] *OED Online, ibid.*

[9] "Arse" is a British rendering of "ass" specific to the latter's vulgar anatomical (as opposed to zoological) meaning. On the ass/arse pun in Early Modern English, see Will Stockton, "'I Am Made an Ass': Falstaff and the Scatology of Windsor's Polity," *Texas Studies in Literature and Language* 49(4), 2007, pp. 340-60.

[10] Glending Olson, "Measuring the Immeasurable: Farting, geometry, and theology in the *Summoner's Tale," The Chaucer Review* 43(4), 2009, pp. 414-427. Timothy D. O'Brien, "'Ars-Metrik': Science, Satire and Chaucer's Summoner," *Mosaic: An Interdisciplinary Critical Journal* 23(4), 1990, 1-22.

[11] Valerie Allen, *On Farting: Language and Laughter in the Middle Ages* (New York: Palgrave Macmillan, 2007).

[12] The preface is unpaginated. This quotation is from the *verso* page beginning "it is thonely thyng (all most) that separateth man from beastes," image 8 in the EEBO scan of the book. On Recorde's dialogue form, see Johnson and Larkey, "Anti-Aristotelian," op. cit. On the contradictions and complexities of dialogue as a pedagogical genre, from Plato onward, see Daniel Boyarin, *Socrates and the Fat Rabbis* (Chicago: University of Chicago Press, 2009).

[13] Henry Cowles, *A Method Only: The Evolving Meaning of Science in the United States, 1830-1910* (Ph.D. Dissertation, Princeton University), on pp. 235-247.

modern technical and philosophical education, informed in part by interpretations of Classical dialogues like Plato's *Meno* where mathematical lessons are imparted by reasoning past initial errors. This early modern pedagogical history suggests a long view of the relationship between algorithms and learning, linked by judgement and reason, rather than opposed in fantasies of mechanism that evacuate expertise from algorithms' operation.

So long a view, it would appear, that in some ways it is circling back on itself: one of the most significant new areas of algorithmic thinking today—data-intensive machine learning—revolves around coaxing digital computers into something very much like what the Master wishes of the Scholar in Recorde's 1543 arithmetic. Eschewing principled, determinate, deductive rules in favor of adaptive and often inscrutable webs of associations,[14] machine learning systems must be trained on a ground set of prepared data and are expected, at least in the beginning, to make basic, sometimes embarrassing, mistakes. If machine learning is the most striking (and perhaps urgent) current parallel to Recorde's didactics, it is hardly the only one. Like the early modern "algorism" that preceded it, more recent modern algorithms derive from habits and principles of procedural learning and practice whose customary naturalization can obscure the essential operation of human agency and error. Notwithstanding modern ambitions for error-free computing through formally rigorous hierarchical design, to code is to tinker, to debug.[15]

Such agency and error are central, even foundational, to Recorde's text. Here, mistakes are generative elements of pedagogy necessary not just to learning but to systematic reasoning. Each error carried specific lessons about reckoning well and general lessons about how to think. Making errors central and productive, rather than exceptional and regrettable, gives a striking and fruitful analytic purchase on algorithms past and present.

Names, Reason, and Authority

The Master and Scholar's exchange about names comes at a pivot in Recorde's opening dialogue. Having convinced the Scholar of the merit of numbering, the Master exchanges with the Scholar a bit of methodological verse. Recorde's books often break into verse, and almost as often—as in the present example—the publisher composed the text as undifferentiated prose. Listen, rather, for the rhyme:

> S. And I to your auctoritie, my wyttes do subdewe: what so ever you say, I take it for trewe. M. That is to moch, and mete for no man, to be beleved in al thynges, without shewynge of reason. Though I myght of my Scoler some credence requyre, yet except I shewe reason, I do not it desyre. (5^{v})

Recorde's dialogues hinge on the Master's exercise of authority, but the Master's authority must be properly derived. The Master's experience becomes evident in response to the Scholar's ignorance, discernment in response to error. The Scholar has to get it wrong before the Master can show how to get it right. No precision without misprision.

The Master's reasoning begins with naming. When the Scholar first resists the Master's question, the latter insists "For greate rebuke it were, a science to have studied, and yet can not tell how it is named" (6^{r}). That is, one must know what one studies. Such knowledge fits one's learning into a systemic whole: here, the lesson on names continues, through naming, into a

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[14] See Matthew L. Jones, "How We Became Instrumentalists (Again): Data Positivism Since World War II," *Historical Studies in the Natural Sciences* 48(5), 2018, pp. 673-684.

[15] Donald MacKenzie, *Mechanizing Proof: Computing, Risk, and Trust* (Cambridge, MA: MIT Press, 2001).

presentation of the "kyndes" or "partes of nombrying" (6^v)—addition, subtraction, and so on—which serves as both a conceptual survey and an outline of the book. Before learning rules and procedures, one must be able to see what they do and how they relate to others. In the Master's words, "Fyrste you muste knowe what the thinge is, and then after learne the use of the same" (7^r). Without that context, arithmetic and algorism lack independent meaning, inscrutable as farts or mumbles.

Readers of Recorde and other early print vernacularizers learn to look closely at word couplings, which typically pair a coinage derived from another language with a more familiar near synonym.[16] The pattern holds for "arithmetike," which according to the Master comes from "arithmos in greke," meaning "nomber" (6^r). The pair in the usual scheme to arithmetic would not be algorism but numbering, which Recorde places at the end of the sentence defining both arithmetic and algorism. Matching arithmetic instead to algorism in the vernacular pair, Recorde did more than just indicate the presence of multiple equivalent terms with distinct etymologies. Arabian algorism's brief mention in *The Ground of Artes* serves at least two specific expository purposes, asserting numbers' novelty and explaining their oddity.

In the chapter on numbering, Recorde suggests both purposes, writing (as Master) "I mought here shewe you , who were the fyrste inventours of this arte, and the reasons of all these thinges that I have taught you, as why you shuld recke[n] your order of places backewarde, I meane from the ryght syde towardes the lefte, with many other thynges, touchynge the causes and reasons of it" (13^v-14^r). Naming the art of numbering with the Arabic algorism, Recorde establishes its place outside of and postdating the Classical sciences. This genealogy gives reason to numbering's peculiar terminology and conventions, such as placing the "first" [smallest] part of a number on the right. At the same time, the attribution situates numbering's users in a distinctly modern relation to Classical knowledge.

The Master never gives the promised explanation of the art's first inventors and the reason for reckoning backward in the first edition of the text. However, the 1558 and subsequent editions include an extended attribution of algorism to "the Chaldays," who "did set these figures as thei set all their letters. For they wryte backwarde as you tearme it, and so doo they reade."[17] The same right-to-left order, the Master notes, "may appeare in all Hebrewe, Chaldaye and Arabike bookes." This satisfies the Scholar, but the Master continues by saying that "the Caldays and Hebrues do not so use their owne numbres," implying (but not stating) that Arabic algorism is the necessary explanation for numbers' ordering. Moreover, the Master adds, English pronunciation from left to right, starting with the largest number, puts the numbers in the same order as the Arabic writing from smallest to largest, right to left. This coincidence settles the matter, and the dialogue continues to the next chapter, Addition.

The work of misprision did not end with the Scholar's initial invocation of awgryme. A second instance reinforces the corrupted term's connection with both the process of learning through correction and the problem of place value in numerical notation. This latter misprision takes place in the chapter on numbering, after the Master asserts that the numeral figure 9, when

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[16] Francis R. Johnson, "Latin versus English: The Sixteenth-Century Debate over Scientific Terminology," *Studies in Philology* 41(2), 1944, pp. 109–135, on 114.

[17] Robert Recorde, *The ground of artes teaching the woorke and practise of arithmetike, both in whole numbres and fractions, after a more easyer and exacter sorte than any lyke hath hytherto beene set forth: with divers new additions* (London: R. Wolfe, 1558), STC (2nd ed.) 20799.5, accessed on Early English Books Online (2017). The pagination in this EEBO edition is unclear; the quoted passage is on image 29 of the EEBO record.

unaccompanied by other figures, always represents the number nine. The Scholar correctly affirms "Then 9 without more figures of Awgrym[18], betokeneth .ix.," but immediately shows his ignorance when, asked to interpret "this example 3679," he matches the figure 7 to the number .vii. (rather than .lxx.) and so on (9^{v}-10^{r}).

Where the Scholar uses the wrong name, it reinforces that the Scholar has also mistaken the concept and usage. As commodious as the figures of algorism will prove in *The Ground of Artes,* they harbored their share of pitfalls and counterintuitive uses. The Scholar's misunderstanding of place value, a problem distinctive to the new notation under discussion, lets Recorde portray the dangers of unthinking number-work. In a very direct sense, the figure's context (in a sequence of figures) determines its meaning (as a numerical value), just as contexts will determine mathematical and other kinds of meaning for the rest of the text. The error shows that even the most apparently transparent instructions are not inevitably applied correctly, that not just their mechanics but also their contexts and rationales must be learned and practiced.

Putting Division in its Place

Long division has long been an iconic, even notorious, part of elementary mathematics curricula. To the extent Recorde's book marked a starting point for such curricula in the English vernacular, it has been there from the start. The procedure's laborious and difficult-to-motivate litany of rules and prescriptions about how to align and manipulate numbers vex and bore students and teachers alike. In recent years, long division has been a lightning rod of sorts for debates about how to teach children to perform arithmetic, think algorithmically, or develop mathematical intuition or creativity.[19]

Recorde's method was not quite the same one most readers of this chapter will have learned in primary school, but it bears a strong family resemblance. Working from the greatest place values to the least, one finds the quotient digit-by-digit by comparing the divisor to the greatest remaining place values of the dividend, subtracting the largest possible round multiple of the divisor from the dividend at each step. The procedure uses the properties of the place value system to break one large division problem into a sequence of simpler problems of division, multiplication, and subtraction.

As in primary school lessons today, this general strategy and its mathematical justification are not spelled out as such in Recorde's text. Algorism was an efficient symbolic means for finding the right answer, not explaining why it was right. Justifications and philosophical principles—including for numerical questions—were matters for geometry, not arithmetic. In Recorde's arithmetical context, understanding was a matter of learning the correct circumstantial application of the method. This, in turn, meant attempting and then learning to avoid incorrect misapplications, and these errors and misprisions show Recorde's sixteenth-century perspective on a centuries-old procedure.

The text's chapter on division follows a set pattern established previously for other basic operations of arithmetic. The dialogue begins with a demonstration by the Master of a simple example, spurred along by queries from the Scholar. The Master marks each step—where to

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[18] Cf. Chaucer's invocation, c. 1390, of "nowmbres of augrym," which Powell ("Welsh Context," op. cit., p. 130) identifies as among the earliest English references to Hindu-Arabic numerals.

[19] E.g. Jay Mathews, "10 Myths (Maybe) About Learning Math," *Washington Post* 31 May, 2005; David Klein and R. James Milgram, "The Role of Long Division in the K-12 Curriculum," 2000, online (accessed 2017), http://www.csun.edu/~vcmth00m/longdivision.pdf; Patricia A. Sellers, "The trouble with long division," *Teaching children mathematics* 16(9), 2010, pp. 516-520.

look, what to calculate, what to write, where to write it, what to call it, and so on—and refers to a sequence of inset figures showing the calculation in progress. Then the Scholar works a more difficult example with interjections from the Master to rectify typical misunderstandings. After these, the sequence repeats with the same numerical examples for a method of checking the outcome of the calculation.

The first error, in fact, tells us more about the text's composition for print than about its author or pedagogical philosophy. The Master begins by explaining how to align the numerals of the dividend and divisor in order to begin the procedure, with the last (leftmost) figures aligned except when the divisor's leftmost figure exceeds that of the dividend, in which case the divisor should be offset by one position to the right. The Master describes the difference by contrasting the quotient of 365 (the number of days of a year) by 28 (the days in a "co[m]men moneth"), by 52 (the weeks in a year), and by 4 (the number of quarters of a year) (62^r). Following the just-described rule, the "2" of 28 would be written under the "3" of 365, whereas the "5" of 52 and the 4 would be written instead under the "6" of 365. These examples involve both of the two possibilities for alignment while opening considerations of possible misunderstandings—for instance, mistakenly applying the rule to the "2" from 52 instead of the "5", mistakenly expecting the "8" of 28 to be smaller than the number above it, or in the shifted case mistakenly expecting 4 to be greater than the number above it after the shift (the "6" of 365). These considerations are not spelled out in the text, however, and the accompanying figures show the 28 erroneously lined up just like the 52, with the "2" under the "6" of 365. Coming in the Master's voice and without comment, this misalignment seems clearly to be a production error (one that remained uncorrected in the 1558 edition) rather than a pedagogical intervention. The alignment appears correctly and without comment on the next page and in the remainder of the calculation (62^v *et seq.*).

The first deliberate misprision comes in the second stage of the Master's example of dividing 365 by 28. Having established 1 as the first digit of the quotient in the first stage, the Master instructs the Scholar to align the divisor of 28 directly under the remainder of 85 from the first stage, so that the "2" of 28 sits under the "8" of 85. Naively following the rule just introduced, the Scholar announces that this 2 in the divisor may be taken 4 times from the 8 in the dividend. "Truthe it is," the Master replies, "that you maye fynde 2 foure tymes in 8," but (turning to the next digits) one cannot likewise find 8 four times in 5 (63^v). Thus, the second digit of the quotient should be 3, not 4, and the Master completes the calculation accordingly. The lesson here: one must look ahead in the calculation and adapt earlier steps accordingly.

Misunderstandings also allow the Master to rule certain considerations as out of bounds or premature. Finding a remainder of 1 after dividing 365 by 28, the Scholar suggests that one might "parte the 1 that remayneth into 28 partes" (64^r). As fractions and proportions had their place in a later part of the book, the Master here replies "That is well sayd, and so must we do in suche cases whe[n] there remayneth any thynge, but I wyll lette that passe nowe, and wyll make you parfecte in hole Divisyon, and wyll here after teache you peculyarly of broken nombre callyd fractions" ($64^{r\text{-}v}$). Proposing to continue the division beyond where the Master would stop, the Scholar shows a misunderstanding not of mathematical principles but of the proper stopping point for a procedure in its conceptual and pedagogical context. The Scholar's misstep previews and begins to justify a later element of Recorde's arithmetic while creating an opening to explain the appropriate place of the current element. The Master's correction, accordingly, asserts a distinction between whole division and fractional arithmetic, reserving the latter for a subsequent exposition.

At this point, the Master hurries the Scholar into the next example. But on recapitulating the result of 365 divided by 28 afterward, another problem surfaces. The Master summarizes the conclusion, "wherby I knowe that in a yere (which contayneth 365 dayes) there are 13 monethes, reckenyng 28 dayes (or 4 wekes) just to a moneth, and 1 day more" (66^{v}). "Why then," responds the Scholar, "do we call a yere but 12 monethes?" To this, the Master replies only that "now it is not convenient to entangle your mynde w[ith] other thynges the[n] do dyrectly pertayne to your mater," and so rules the question outside the scope of the lesson (66^{v}-67^{r}). The Scholar's insight that the arithmetical calendar invoked in the example contradicts the usual calendar becomes, here, a different kind of error: expecting a mathematical reason for an anomaly with non-mathematical causes (history, astronomy, religion, and so on). When learning arithmetic, the Master instructs the Scholar, one must keep to things that are arithmetical, and recognize when the answer you seek might require other kinds of knowledge and reasoning.

Rules without Practice

The second worked example of long division, with the Scholar at the helm, offers more opportunities for mistakes. The Master sets the problem of dividing 136280 by 452, and the Scholar commits a first mistake from the very moment of transcribing the numbers in question. Having set the "4" of 452 under the "1" of the dividend, the Scholar—when prompted by the Master—admits that "I had forgotten" to "sette the dyvisor one place more forward towarde the ryght hande," since 4 exceeds 1 (64^{v}). Following the earlier printing error, this is the first purposeful mistake regarding alignment, signaling this as a notable sort of error for the proper use of place-value. The Scholar admits to forgetting so that the Master can insist on the importance of assessing each step and remembering adjustments and exceptions in place-valued arithmetic.

The Scholar completes the first stage of the division without further incident, but encounters a hitherto untreated exception when setting up the next stage. By pedagogical design, the Scholar finds upon shifting the divisor by one place to start the next stage that all the numerals above the leading digit of the divisor have already been cancelled out in the first stage. The Master explains that in such situations one must "write in the quotie[n]t a cyphre 0" and adds "The reason of this wyll I showe you hereafter" (65^{v}). Though the explanation is deferred, two lessons are clear. First, when setting up a stage of the division one does not simply move incrementally to the right each time; rather, one must continue to look at the value remaining from the dividend. Second, as in other operations of arithmetic, ciphers index shifts in place value, a point the Master reinforces by instructing the Scholar to place a cipher for every time such a shift is needed.

The Scholar continues the calculation, eliciting a brief suggestion from the Master that taking 5 from 28 to leave 23 could just as well have been described as taking 5 from 8 to leave 3, with the "2" in the higher place unchanged. Stuck at the last step, which amounts to subtracting 2 from 230, the Scholar asks the Master "what shal I nowe do?" The answer is to "do as you learned in Subtraction in a lyke case" and borrow 1 from the 3 in the second place (66^{r}). Having successfully applied the principles of one kind of arithmetic to another, the Scholar completes the division and the Master summarizes the result (66^{v}).

One successful quotient alone did not teach the method, however. The Master instructs the Scholar that "you have lerned a shorte maner of dyvisyon, whiche I wolde have you often to practyse, so that you may be parfecte in it" (67^{r}). Here, the Master evokes an earlier invocation from the chapter on Numeration:

> I wyll yet exhorte you now, to reme[m]ber both this, that I have said, and all that I shall saye, & to exercyse your selfe in [the] practise of it: for rules without practise, is but a lyght knowledge, and practise it is, that maketh men p[er]fecte and prompte in all thynges. (14^{v})

It lacks the pith of the adage that practice makes perfect, but the message is unmistakable. In *The Ground of Artes,* just knowing the rules without having perfected their application through repeated practice can only be a light, superficial knowledge. The exposition bears this out repeatedly: simply following a rule, failing to place it in its conceptual and procedural context, leads again and again to mistakes and misunderstandings. Algorithmic thinking required the discerning execution of methods learned and practiced at length, with skill and judgement.

With rule-based calculation so evidently error prone, Recorde has the Scholar worry about how to “examyne and trye my worke, whether I have done well or no,” without an expert on hand to offer corrections (67^{r}). “Sume men (yea and co[m]menly) do trye that,” the Master offers, “by the rule of 9,” suitably adapted to division. The rule of 9 first appears in the chapter on Addition, when the Master instructs the Scholar that “it is often practise that maketh a man quyke and rype in all thynges. But bycause of suche greate summes, in whiche there may chau[n]ce to be some errour, I wyll teache you, how you shall prove whether you have done well or no” (23^{v}). (Here and elsewhere, “prove” and “proof” have the sense of a trial or test—as in “the proof of the putting is in the eating” or “proof-reading”—not, as typically used in mathematics, of a rigorous demonstration of certain validity.) The procedure involved treating all the numerals in the calculation “as though they were all unities,” ignoring place values and adding them together as units. Whenever the sum “encreaseth above 9, cast away 9” and continue. If the summands and the sum yield the same result after such a calculation, “then have you well done, but yf they be unlyke, then have you myssed” (24^{r}).

A brief digression into twentieth-century mathematics may help account for the method’s principles and limitations. In anachronistic terms, the procedure applies the canonical surjection from the ring of integers to the ring of integers modulo 9. Since this is a ring homomorphism and $10 \equiv 1 \bmod 9$, this amounts to adding the digits on both sides of the equation and reducing modulo 9 as one goes. The method allows one to catch an error of addition provided that the net of all errors in all the digits of the calculation is not itself a multiple of 9. The method is not as uniformly effective for division problems since there are more ways for errors to compound as multiples of 9—at one extreme, the method says nothing about the quotient when the divisor is itself divisible by 9—but it was certainly quicker than the alternative of checking directly by multiplication, and mostly reliable. Such considerations, however, did not enter into the dialogue in *The Ground of Artes*.

To check division with the rule of 9, the Master revives the cross, a device introduced in the Addition chapter for checking sums involving multiple denominations (e.g. of currency; 34^{v}-35^{r}).[20] Angled in an “x” shape, the cross displays numerals as two pairs—left and right, top and bottom—for ready manipulation and comparison. Here, the cross organizes the four constituents of division: the dividend, divisor, quotient, and remainder, the latter implicitly by way of an intermediate calculation. Keeping to a third-person description of what “sume men … do trye,” the Master describes first casting out nines from the quotient and divisor, noting the results on the left and right of the cross respectively. The Master then multiplies the results together and

[20] Here the Master teases the Scholar when the latter asks if the method works equally well for small currencies like halfpennies, farthings, kewes (eighth-pennies, denoted “q”), and cees (sixteenth-pennies, denoted “c”): “You thynke you be at Oxforde styll, you brynge forthe so faste your q, and c” (36^{r}).

adds the remainder from the division, casts out nines again, and places the result at the top of the cross. Upon casting nines from the dividend and noting the result at the bottom of the cross, one passes the test if the top and bottom of the cross agree. Significantly, after the Master demonstrates the method by checking the result of 365 divided by 28, the Scholar then performs the corresponding check for the calculation of 136280 divided by 452 without mistake.

Or rather, without computational mistake. When the Scholar proudly declares that with the rule of 9 "I knowe that the division was well wrought," the Master admonishes that the rule is only "the co[m]men profe" and that "the more certayne workying" would require inverting the calculation with a full process of multiplication (68^{v}). The Scholar's error, here, is one of overconfidence in common tests in place of those that, while more cumbersome, are also more certain. Even when one has followed rules correctly, one must know what those rules mean and what they imply for subsequent assessments and calculations.

Farting for Modernity

Which brings us back to arses. That is, to the relationship between naming, mistaking, and algorithmic thinking in early modernity. Recorde presented algorism in 1543 as a fruitful and dynamic approach to solving mathematical problems, one requiring dedicated practice and experience to execute well. Learning to think algorithmically required acquiring habits of rule following premised on each rule's sensitive dependence on context and judgement. Context and judgement, in turn, hinged on naming, of knowing whereof one spoke. That is why, in *The Ground of Artes,* the commodities and names of the various parts of arithmetic came before the practice of those parts.

Before getting even arithmetical names right, it was necessary to get them wrong. Algorithmic reasoning was creative, alive with possibility. Misprision marked that creativity pedagogically by demonstrating the possibility of miscreating while validating the importance of experience and offering a means to attain it. Through mistakes, adaptations, practice, and discernment, one could learn to recognize and resolve the multifarious configurations of numerical arithmetic. Getting words wrong was thus a prelude to getting numbers and procedures wrong, and all of these kinds of mistakes showed how to get arithmetic right. Recorde's dialogical exposition proceeded by error and correction because this was the essence of arithmetic, its source of power.

Writing in dialogue, Recorde eschewed the other dominant form of early modern mathematical exegesis: the orderly and methodical deduction associated with Euclidean geometry. Indeed, Recorde's own 1551 book of Euclidean geometry, *Pathway to Knowledg*, kept to the Euclidean mode of sequential constructive exposition even as it subverted other elements of the genre.[21] For most early modern authors, Classical geometry stood, to varying degrees of explicitness, in constant contrast to the newer Arabic mathematical arts of algebra and algorithm.[22] Arithmetic, associated not just with post-Classical civilization but also the worldly

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[21] Robert Recorde, *The pathway to knowledg, containing the first principles of Geometrie,as they may moste aptly be applied unto practise, bothe for use of instrumentes Geometricall, and astronomicall and also for projection of plattes in everye kinde, and therfore muche necessary for all sortes of men* (London: R. Wolfe, 1551), STC (2nd edn.) 20812, accessed on Early English Books Online and in the Cambridge University Library. See Barany, "Translating Euclid's Diagrams," op. cit.

[22] On this contrast, which remained a key to interpreting mathematical discourse at least through the nineteenth century, see Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford: Oxford University Press, 1996), ch. 1-3; Michael J. Barany, "God, King,

preoccupations of commerce and money,[23] was always overtly or implicitly a departure from the stale intellections of Ancient minds.

Pedagogical dialogues, by their very construction, are polyphonous texts, bringing together multiple forms of speech. Valerie Allen indicates that the broken, profane, multisensory expression of farting was very much a part of late medieval and early modern conceptions of language.[24] Farts linked to laughter, puns, incantations, and other linguistic productions that sat opposite the reasoned forms of trivial grammar and rhetoric. There were, in the period’s rhetorical, medical, and alchemical theory and discourse, multiple meaningful ways to expel air.[25] In its hybridity, excess, and alienation, the boundary-troubling experience of farting marks, in Allen’s analysis, a mode of transition for modernity itself, one she compares to the transition from trapped wind to passed gas via the anal threshold and bifurcated cheeks.[26]

Chaucer’s ars-metrike sat on this kind of epochal precipice, parodying number-obsessed early modernizers like the so-called Oxford calculators of Merton College.[27] The ontology and epistemology of geometrical partition in the *Summoner’s Tale*—situated in the devil-inspired construction problem “That every man sholde have yliche his part / As of the soun or savour of a fart?”[28]—implicated fundamental questions about the nature and divisibility of numbers and measures.[29] A close reading of Chaucer’s ars-metrike, in this light, places this farting arithmetic at the cusp between an older model that sought to resolve these questions through Classical and canonical authority and an emerging model that favored experiment and experience.[30]

This understanding of Chaucer’s ars-metrike and its more-explicit and better-studied mass of allusions allows a more capacious interpretation of Recorde’s Scholar’s misprision of arithmetic as arsmetrike. If Classical geometry united head and hand—reasoning and drawing—in a robust authoritative tradition, early modern arithmetic found its most honest expression in a baser organ, with its characteristically modern manner of speech. Recorde makes a virtue of the Scholar’s persistent penchant for mistaking, forgetting, and misapplying—not quite speaking in farts, but perhaps speaking after the manner of farting. These misprisions give an experiential basis for knowledge that can penetrate beyond the “light knowledge” of rote procedure.

In the century after Recorde, playwright Ben Jonson gives the vain and oft-mistaken Sir Glorious Tipto in *The New Inne* (1631) the line:

Fart upon *Euclide,* he is stale, & antique,

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and Geometry: Revisiting the introduction to Cauchy’s *Cours d’analyse,*” *Historia Mathematica* 38, 2011, pp. 368-388, on 372-379.

[23] See Joel Kaye, “Money and Administrative Calculation as Reflected in Scholastic Natural Philosophy,” in David Glimp and Michelle R. Warren, eds., *Arts of Calculation: Quantifying Thought in Early Modern Europe* (New York: Pagrave Macmillan, 2004), pp. 1-18.

[24] Allen, *On Farting*, op. cit.

[25] O’Brien, “Ars-Metrik,” op. cit., esp. 2-8.

[26] Allen, *On Farting,* op. cit., 2.

[27] Olson, “Measuring the Immeasurable,” op. cit., 415-419; Kaye, “Administrative Calculation,” op. cit.

[28] Geoffrey Chaucer, *The Canterbury Tales,* ed. F. N. Robinson (1957), University of Michigan Corpus of Early English Prose and Verse, https://quod.lib.umich.edu/c/cme/CT/1:3.6?rgn=div2;view=fulltext, accessed 2019, III: 2222-2223.

[29] Olson, “Measuring the Immeasurable,” op. cit.; O’Brien, “Ars-Metrik,” op. cit.; Amir Alexander, “The Imperialist Space of Elizabethan Mathematics,” *Studies in the History and Philosophy of Science* 26(4), 1995, pp. 559-591.

[30] O’Brien, “Ars-Metrik,” op. cit., 18.

Gi’me the modernes[31]

If the great mathematical minds of antiquity spoke in geometry, modernity belonged to the arses that farted algorism. Only in the seventeenth century, when the calculator-mad likes of Leibniz enrolled arithmetic in their fantasies of mechanical decidability, did algorithms begin to lose the scent of creativity and indeterminacy. Even then, philosophers and artisans entertained a tremendous variety of views on the relationship between mechanism and creativity.[32]

Across this history, formally deterministic mechanical rationality has always been the stuff of principles rather than practices. It is one thing to have rules and a system for counting to a billion, and quite another to carry it off with fingers and toes, words and tallies.[33] Discharge the task to a machine—say, the prolifically counting electronic computers that drive twenty-first century digital economies—and the result is more mediation and more credence in the unseen operations of counters that further defer human accountability. One way to understand modernity is in terms of a widening and deepening willingness to credit in-principle abstractions over context-laden practices.

Recorde’s early modernity, by contrast, was one where principles were intimately bound up in practices and contexts. If algorithms seem now to evacuate judgement rather than demand it, that is only because the early modern embrace of algorism has been joined by a later modern move to write misprision out of calculation’s essential operation. Even where context and contingency have been embraced as essential to the practice of reasoning, formalist fantasies have sterilized such practices of their error-bound roots. Nicolas Bourbaki, modern prophet of rigorous logical structures, called for mathematicians to exercise “experience and mathematical flair” in place of tedious idealized deterministic deductions.[34] For Bourbaki, a high modern confidence in the structural guarantees of in-principle calculation justified informal reasoning’s predominance for working mathematicians, with careful delineations of context ensuring the rigorous validity of playful reasoning.[35]

As modernity’s critics have recognized under many different rubrics, such a divorce of error from understanding, of mistake from calculation, was no small feat.[36] Calculation is an enterprise replete with misprision, from the schoolroom to the electronic computer database and beyond. Where Recorde’s narrative performs the imbrication of mistake and reason, later

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[31] Ben Jonson, *The New Inne* (London: Thomas Harper, 1631), STC (2nd ed.) 14780, EEBO, Act II, Scene 5.

[32] Matthew Jones, *Reckoning with Matter: Calculating Machines, Innovation, and Thinking about Thinking from Pascal to Babbage* (Chicago: University of Chicago Press, 2016).

[33] See Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics,* trans. G.E.M. Anscombe, eds. von Wright, Rhees, and Anscombe (Oxford: Basil Blackwell, 1956); Michael Lynch, “Extending Wittgenstein: The Pivotal Move from Epistemology to the Sociology of Science,” in A. Pickering, ed., *Science as Practice and Culture* (Chicago: University of Chicago Press), pp. 215-265. Tallies and finger-counting would play a pivotal role in late-nineteenth century debates about the origins and meaning of modernity; Michael J. Barany, “Savage Numbers and the Evolution of Civilization in Victorian Prehistory,” *British Journal for the History of Science* 47(2), 2014, pp. 239-255.

[34] Nicolas Bourbaki, *Elements of Mathematic: Theory of Sets* (Paris: Hermann, 1968), 7-8.

[35] Michael J. Barany, “Integration by Parts: Wordplay, Abuses of Language, and Modern Mathematical Theory on the Move,” *Historical Studies in the Natural Sciences* 48(3), 2018, pp. 259-299.

[36] E.g., among many explicitly to consider determinist fantasies of formal calculation as symptomatic of modernity’s mystifications, Max Horkheimer and Theodor W. Adorno, *Dialectic of Enlightenment: Philosophical Fragments,* trans. Edmund Jephcott, ed. Gunzelin Schmid Noerr (Stanford: Stanford University Press, 2002).

pedagogies relegate mistake to unwritten disciplinary byways of training, debugging, accidents, and exceptions. But that imbrication is precisely what makes modernity’s reason fruitful. Fart upon stale and antique philosophies and historiographies of deterministic algorithmic thinking! In early modern arithmetic, we can recognize that algorithmic modernity itself called for new, unruly ways of speaking, and different organs of reason.