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On Remediation: Media, repair, and the discipline of fantasy in the theory and practice of algorithmic modernity.

by Michael J. Barany*

ABSTRACT

Nothing ever works quite as imagined. This truism, based in fundamental features of both work and imagination, raises vital questions about mediated metaphors and practices of enacting and enforcing theoretical fantasies about algorithmic systems and systematic mathematical knowledge. Methods of moving conceptual figurations between media—such as blackboards or printed paper—as well as explicit and implicit ideas about such transmutation in conceptual practice, suggest the thoroughgoing importance of remediation: repair, correction, backfilling, but also (as a pun) patterns of mediation and transit across material manifestations. Such remedial work links algorithmic practice and fantasy from early modernity to the present, from the elementary to the elite. Remediation appears in deliberate theories and practices of error detection and correction, as well as in habituated modes of conceiving of and intervening in large conceptual systems characteristic of modern algorithmic and mathematical thought and action. I explore these here episodically, elaborating an account of remediation and algorithms through analyses of sixteenth century arithmetic, nineteenth century blackboard practice, and twentieth century efforts to establish global institutions and universal foundations, concluding with two twenty-first century episodes that illustrate some present stakes of this analysis.

* Science Technology and Innovation Studies, University of Edinburgh, U.K.; michael@mbarany.com

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INTRODUCTION: BACK TO BASICS

For centuries, remedial arithmetic has been at the forefront of mathematical theory and algorithmic practice. This conclusion is both underscored and obscured by the sheer ubiquity of arithmetic in the modern world, from the schoolroom to the marketplace and beyond. Remedial arithmetic is pervasive because arithmetic is pervasive, and because this supposedly routine and reliable algorithmic practice never quite holds up as routinely or reliably as it is supposed to. The massive modern infrastructures of inculcating, routinizing, mechanizing, and continually correcting all-too-errant arithmetic testify to the immense social, technological, and cultural achievement of ubiquitous arithmetic in the modern world, even as those infrastructures' success makes it harder to see just what an immense achievement ubiquitous arithmetic is.[1]

Today's narrow conventional conception of remedial arithmetic, trained on the repair of deficient learners,[2] is conditioned on a long history of seeing arithmetic itself as broken and of proposing to repair it. Remedial arithmetic in this wider historical sense encompasses challenges to the ontology and epistemology of numbers and calculation significantly predating the age of algorithms and continuing into the present. Numbers and their manipulation have been enduring sources of philosophical paradox and technical consternation. That centuries of challenges and remedies to arithmetic practice sit just behind a commonplace image of the "so very simple" figuring of schoolchildren is not a contradiction.[3] It is, rather, a manifestation of the powerful interplay of figuring and of reasoning about figuring that makes algorithmic modernity possible. The project of remedying arithmetic, and of casting it as in need of remedy, has been the quiet vanguard of thinking and calculating for as long as these activities have been, in one sense or another, algorithmic.

A fundamental challenge for accounts of algorithms in the modern world is to understand the relationship between how people experience and interact with algorithms and how people imagine algorithms to work. Often this is a relationship of misalignment,

[1] By this token, arithmetic fits Star and Ruhleder's (1996, 113) multidimensional definition of infrastructure, as an embedded, transparent, extensive, membership-conferring, practice-linked, standard-embodying, built-up configuration most visible upon breakdown; Susan Leigh Star and Karen Ruhleder, "Steps Toward an Ecology of Infrastructure: Design and Access for Large Information Spaces," *Information Systems Research* 7 (1996): 111-134.

[2] The focus on learners' deficiencies is often closely associated with learners' marginalized identities. See Danny Bernard Martin, "Equity, inclusion, and antiblackness in mathematics education," *Race Ethnicity and Education* 22 (2019): 459-478; Sarah Theule Lubienski and Andrew Bowen, "Who's Counting? A Survey of Mathematics Education Research 1982–1998," *Journal for Research in Mathematics Education* 31 (2000): 626-633.

[3] I refer to Tom Lehrer's satirical 1965 characterization of an arithmetic method from the controversial "New Math" school curriculum as "so very simple that *only* a child can do it." The New Math was an unusually explicit recent manifestation of the politically and epistemically charged links between school reckoning and elite mathematical theory. See Christopher Phillips, *The New Math: A Political History* (Chicago, 2014). One must take care to historicize the commonplaceness of schoolroom arithmetic. If, as Stedall posits, in modern societies schoolchildren typically far outnumber adults among those doing mathematics at any given time, it must be remembered that this condition is *characteristically* modern, dependent on historically recent pedagogical modes and expectations; Jacqueline Stedall, *The History of Mathematics: A Very Short Introduction* (Oxford, 2012), ch. 4.

frustration, disappointment, or error. Algorithms abound, thrive, portend, and inspire because often this is also a relationship of repair, accommodation, forgiveness, projection, or hope.[4]

This chapter offers a theory of algorithmic knowledge and practice based on media and remediation, on the material and imaginative contexts of repair. I begin with an orientation to perspectives from social studies of mathematics and media that help to frame my historical analysis of algorithmic thought and action. I then briefly historicize elementary arithmetic, once nearly synonymous with algorithm, identifying it as a locus of practical and philosophical engagement with concerns central to the history of algorithms.

The remainder of the chapter takes a kaleidoscopic approach, starting with an early modern method of summing and error-catching and finishing with twentieth-century proposals to rectify the very most elementary facts of arithmetic. In between, I elaborate the material and epistemic relationships between theory, practice, and repair by counterposing two mathematical media: the blackboard, a medium of pedagogy and research defined in large part by its situated immobility, and the abstracting journal, a medium of elite mathematical research intended to make new mathematics mobile. These intermediate sections connect the analysis of elementary arithmetic to other forms of systematic knowledge and practice, demonstrating how an analysis of remedial arithmetic, in the sense developed here, can provide a broader account of algorithmic modernity. The conclusion returns to the frontiers of mathematical research and the tension and ambivalence that accompanies the materiality and sociality of remediation in the negotiation of formal knowledge.

Remedial arithmetic represents the perspective that algorithmic practice is perpetually an unfinished, imperfect, error-laden attempt at algorithmic ideals. I argue that mistake and failure, rather than contradict algorithmic systems' fantasies of order and coherence, are the very conditions of those fantasies. Remediation is the conduit from partial, provisional knowledge and its institutions to projected, complete systems. To remedy a fault is to render it inessential, to conjure the possibility of faultlessness. Errors become opportunities for repair that reinforce those systems' integrity, creating discretionary opportunities to define what integrity might mean.[5] Remediation is creative, dynamic, and constitutive, making ideals into resources for practice and making practice into a representation of ideals.[6]

Time and again, such remediation takes place through specific acts of re-mediation, translating representations from one material medium to another. Following errors and remedies across media and identifying media's implication in and implications for processes of remediation, this chapter suggests a method and perspective to examine the relationship between formal and ideal knowledge and its cultural and experiential settings and conditions. Though errors and exceptions are, by construction, exceptional, their means and conditions of

[4] Stevens (this volume) characterizes this phenomenon in terms of the history of "vaporware" and the creative gaps between vision and working code.
[5] See Passi and Sengers, "Making data science systems work," *Big Data and Society* 7 (2020).
[6] See Stephanie Dick, "The Politics of Representation: Narratives of Automation in Twentieth Century Mathematics," in Morgan, Hajek, and Berry, *Narrative Science* (Cambridge, forthcoming).

remediation are thoroughly integral to the orderly pedagogical and theoretical production of modern ways of knowing and doing.

Media Theories and Remedial Practices

The long history of algorithmic thought and practice is filled with a staggering variety of media, as varied as inked numerals in rows and tables, manuscript and printed books and their annotations, slates and blackboards, scrap paper, journals, microfilm, postcards, electronic computers, and digital networks, each with their associated infrastructures, technologies, habits, and traditions. That so many substrates should all support their own forms of algorithmic knowledge is staggering. That they should all support *the same* such knowledge demands a reckoning with knowledge, media, and the production of sameness alike.

Media theories of modernity abound, relating technical means of reproducing and sharing representational media and social means of reproducing and sharing identities, politics, economies, and cultures.[7] Shared and distributed modes and technologies of writing and reading, creating and copying, publishing and pirating, teaching and learning, have made it possible to create functioning collectivities and institutions at large scales and to imagine discursive, political, and other communities at even larger scales. The same generalization holds for theories as for people: the scope and scale of theoretical and technical systems has depended on media of assembling and distributing and coordinating representations of theories as well as of imagining systems of knowledge and method that extend in principle far beyond what any individual or substantially interacting community can represent in practice.[8]

The difference between the immediate and the imagined matters. Accounts of the constitutive gaps between practices and ideals have long been foundational to the social study of knowledge, and algorithmic knowledge is no exception.[9] With remedial arithmetic, this gap is what lets a child attempting (and often mistaking) sums in a specific time and place act as though summing were universal, notwithstanding the tremendous known variation in

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[7] E.g. Elizabeth L. Eisenstein, *The Printing Press as an Agent of Change* (Cambridge, 1980); Marshall McLuhan, *Understanding Media: The Extensions of Man* (McGraw-Hill, 1964); Benedict Anderson, *Imagined Communities: Reflections on the origin and spread of nationalism* (Verso, 1983); Bruno Latour, *We Have Never Been Modern,* trans. Catherine Porter (Harvard, 1993); Adrian Johns, *The Nature of the Book: Print and Knowledge in the Making* (Chicago, 1998). On media and reproduction, see also Jay David Bolter and Richard Grusin, *Remediation: Understanding New Media* (MIT, 2000); Bolter and Grusin briefly mention repair and remedy but instead emphasize other dimensions (in etymology and pun) of remediation in their theory of visual representation, including (as here) how newer media recapitulate older media.

[8] On scale, reason, and infrastructure, see Paul N. Edwards, *A Vast Machine: Computer Models, Climate Data, and the Politics of Global Warming* (MIT, 2010); Anna Lowenhaupt Tsing, “On Nonscalability: The Living World Is Not Amenable to Precision-Nested Scales,” *Common Knowledge* 18 (2012): 504-524; Tarleton Gillespie, “Content Moderation, AI, and the Question of Scale,” *Big Data and Society* 7 (2020). On representation and theory, see Coopmans, Lynch, Vertesi, and Woolgar, eds., *Representation in Scientific Practice Revisited* (MIT, 2014). Csiszar (this volume) discusses the significance of the scale of imagined infrastructures in science metrics.

[9] E,g, David Bloor, *Knowledge and Social Imagery* (Routledge, 1976); Donald MacKenzie, *Mechanizing Proof: Computing, Risk, and Trust* (MIT, 2001).

summing across and within human societies.[10] Even for the same individual, counting to ten and counting to ten thousand are radically different acts in practice, and must be learned to be seen as the same in principle. Counting to ten million the same way one counts to ten is not humanly possible by virtually any accounting, except that of the idealized principle of counting. Such differences can even function as evidence of underlying universality: the more different things can be rationalized as ultimately the same thing, the more universal that ultimate same thing must be.[11]

Crucial to training in arithmetic is learning to see the same thing in this diversity of practices, to excuse and ignore and remedy those people and practices not so aligned. A trained calculator understands differences in calculations as personal and reparable exceptions rather than as challenges to the unity and integrity of the calculation and the system in which it is embedded.[12] Comprehensible examples with surveyable and verifiable elements stand in for a universe of potential calculation that cannot be directly represented or affirmed.[13] Try to multiply two forty-digit numbers on paper with the same method you use for three-digit numbers; now try to convince yourself you have done it correctly. Classrooms of students reproducing authoritative inscriptions from a common blackboard or textbook learn to understand their bookwork as an imperfect example of a uniform ideal against which they can be examined and measured.[14] They become part of an idealized homogeneous community of classroom calculators, in turn part of a much larger imagined community that would in principle get the same result from the same calculation.

In this empire of calculation, reckoning can be parceled, divided, passed along hierarchies, assembled, and transmitted, made fungible like commodities or rarified like luxury crafts.[15] Calculation is an institution. The gestalt effect of a system of calculation is a product of partial mutual understanding and a learned willingness to believe that such parts add up to a whole.[16] The lived experience of mathematical theory, code, and algorithms is at

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[10] Eleanor Robson, *Mathematics in Ancient Iraq* (Princeton, 2008); Jean Lave, *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life* (Cambridge, 1988), Michael J. Barany, "Savage Numbers and the Evolution of Civilization in Victorian Prehistory." *The British Journal for the History of Science* 47 (2014): 239–55.

[11] See Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications in Pure and Applied Mathematics* 13 (1960):1-14; Ivor Grattan-Guinness, "Solving Wigner's Mystery: The Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences," *The Mathematical Intelligencer* 30 (2008): 7-17; Ian Hacking, *Why Is There Philosophy of Mathematics At All?* (Cambridge, 2014), chs. 3, 5, 6.

[12] Mathematicians' resilience in excusing and managing exceptions, faults, and contradictions is a central motif in Imre Lakatos, *Proofs and Refutations* (Cambridge, 1976).

[13] Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics* (Blackwell, 1956); Eric Livingston, *The Ethnomethodological Foundations of Mathematics* (Routledge, 1986); Brian Rotman, *Ad Infinitum… The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back In. An Essay In Corporeal Semiotics* (Stanford, 1993); Donald MacKenzie, "Slaying the Kraken: The Sociohistory of a Mathematical Proof," *Social Studies of Science* 29 (1999): 7-60; Michael J. Barany, "Of Polyhedra and Pyjamas: Platonism and induction in meaning-finitist mathematics," in De Freitas, Sinclair, and Coles, eds. *What is a Mathematical Concept?* (Cambridge, 2017), 19-35. See Steingart (this volume) on the rhetorical achievement on transparency and simplicity for purportedly basic mathematics.

[14] Michael J. Barany and Henry Cowles, "Thirteen Ways of Looking at a Blackboard," forthcoming.

[15] Lorraine Daston, "Enlightenment Calculations." *Critical Inquiry* 21 (1994): 182-202.

[16] See Michael J. Barany, "Integration by Parts: Wordplay, Abuses of Language, and Modern Mathematical Theory on the Move," *Historical Studies in the Natural Sciences* 48 (2018): 259-299..

least as shaped by managing incomprehension and misunderstanding as it is by sharing conviction and certainty.[17] Philosophers' and mathematicians' overwhelming focus on the latter has dominated the mathematical historiography to the effect of misleadingly naturalizing the immensely difficult social achievement of coherence and universality.[18]

Reemphasizing processes of following, rederiving, correcting, and muddling-through returns mathematical media to the center of the story. Following the epistemic practices of contemporary mathematicians, I have described an intimate connection between remediation and the process of translating mathematical representations from one medium to another through a dynamic of writing up and reading down.[19] Writing up involves formalizing a finding in media meant to move, such as typeset articles or lecture notes. Mathematicians tend to discount the epistemic labor of writing up, downplaying just how hard it is to get results into a form that travels well. On arrival, these results can be hard to interpret, and mathematicians go through an inverse process I have called "reading down" to understand and redeploy written-up results. Reading down happens on comparatively immobile media, such as blackboards or scraps of paper. On these, mathematicians preserve a dematerialized ideal of the knowledge they produce by correcting the record in front of them—smudging and crossing out and rewriting and seeing each specific materialization as provisional and inessential. In this light, the unstable, partial, mediated nature of theoretical coordination is not a counterpoint to theoretical knowledge's in-principle universality but fundamentally constitutive of it.[20] Remediation as repair and as material transformation explains how plural practices can underwrite unitary principles.

THE PERSISTENCE OF REMEDIAL ARITHMETIC

Arithmetic used to seem far less unitary. In its Medieval and Early Modern incarnations, in the handwork and tablework and discourses of merchants and tradespeople and scholars, algorithm and arithmetic referred somewhat interchangeably to a constellation of methods for figuring with numerals.[21] Representing numbers with numerals—decimal symbols like 3, 5, and 8—was not a common or obvious thing to do. Neither was repositioning, scratching out, and rewriting these into other numeral representations according to routinized rules to perform a calculation. Indeed, early modern textbooks such as the one discussed in the next section deliberately distinguished between numbers and the strange numerals being used to

[17] See Michael J. Barany, "Organizational Practice in the Heterolingual Archive [Prática organizacional no arquivo heterolingual]," *Em Construção* 7 (2020): 19-27.
[18] See Chemla, Chorlay, and Rabouin, eds., *The Oxford Handbook of Generality in Mathematics and the Sciences* (Oxford, 2016) on the diverse manifestation, production, and valuation of generality.
[19] Michael J. Barany and Donald MacKenzie, "Chalk: Materials and Concepts in Mathematics Research," in Coopmans *et al*, eds., *Representation,* op. cit., 107-129.
[20] On rematerialization and algorithmic media, see Jacqueline Wernimont, *Numbered Lives: Life and Death in Quantum Media* (MIT, 2019).
[21] This chapter focuses on the early modern and modern history of arithmetic and algorithm stemming from this etymological point of departure. There is a rich literature on related themes including algorithms, calculation, repetition, and the production of generality and universality in other contexts, e.g. Karine Chemla, ed., *The History of Mathematical Proof in Ancient Traditions* (Cambridge, 2012), esp. Chemla, "Reading proofs in Chinese commentaries: algebraic proofs in an algorithmic context," ibid., 423-486.

represent them, taking care to consider numeral manipulations as a (frequently unreliable) proxy for the mathematics of the numbers in question.

Far from the hegemonic, ubiquitous practice of school drill and routine tabulation, figurate arithmetic was a new, exciting, even transgressive development in the annals of calculation that coexisted more visibly with other species and technologies of number work recognized as convenient or advantageous for different purposes.[22] Abacuses, counting boards, corporeal mnemonics, longhand records, and other varieties of writing and reckoning were, in the right settings, easier, faster, more reliable, more familiar, more tamperproof, or otherwise preferable to numeral reckoning. Early texts touted the multifarious advantages, both theoretical and practical, of the new numeracy. In treatises associated with names like al-Khwarizmi and Fibonacci that indexed expansive and often-unpreserved histories of sharing and adapting and thinking about figuring practices,[23] these methods appear as remedies for hands and minds, as simplifications and improvements to the art and science of calculation.

What started as a remedy for the trouble of calculation quickly became troublesome in its own right. Rules met with exceptions. Rote procedures that led swiftly to the right answer in some contexts threw up paradoxical apparitions in others: negative, impossible, irrational, imaginary, departed, and other vexing quantities have been the targets of centuries of calls for repair and reform to the rudiments and architectures of reckoning.[24] The deceptively fundamental-seeming basic elements and operations of arithmetic have long taunted philosophers, engineers, and all manner of reformers and revolutionaries of human thought.[25] The history of arithmetic and its remedies has itself been cast as evidence of the progress and perfectibility of men's minds and symbolic reasoning, [26] or of minds' and reasoning's ultimate irremediability.[27]

Since algorithm and arithmetic were effectively synonymous for much of their joint early history, this broken, troublesome, patched-back-together history of arithmetic is also historically fundamental to algorithms, returning and crossing and tangling even after their always-incomplete etymological divergence. The circuitous etymology of algorithm itself carries these features. From ambiguous roots predating the modern languages in which

[22] Reviel Netz, "Counter culture: Towards a History of Greek Numeracy," *History of Science* 40 (2002): 321-352; Jessica Otis, *By the Numbers* (Oxford, forthcoming).
[23] See Jens Høyrup, "Leonardo Fibonacci and *abbaco* culture: a proposal to invert the roles," *Revue d'Histoire des Mathématiques* 11 (2005): 23-56.
[24] E.g. Gert Schubring, *Conflicts Between Generalization, Rigor, and Intuition: Number Concepts Underlying the Development of Analysis in 17th-19th Century France and Germany* (Springer, 2005). Leo Corry, *A Brief History of Numbers* (Oxford, 2015).
[25] Matthew Jones, *Reckoning With Matter: Calculating Machines, Innovation, and Thinking about Thinking from Pascal to Babbage* (Chicago, 2016); Ivor Grattan-Guinness, *The Search for Mathematical Roots 1870-1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel* (Princeton, 2001); Erickson et al., *How Reason Almost Lost Its Mind* (Chicago, 2013), esp. ch. 1; Donald MacKenzie, "Negotiating Arithmetic, Constructing Proof: The Sociology of Mathematics and Information Technology," *Social Studies of Science* 23 (1993): 37-65.
[26] Kevin Lambert, "A Natural History of Mathematics: George Peacock and the Making of English Algebra," *Isis* 104 (2013): 278-302; Barany, "Savage Numbers," op. cit.
[27] Paolo Mancosu, "Between Vienna and Berlin: The Immediate Reception of Gödel's Incompleteness Theorems," *History and Philosophy of Logic* 20 (1999): 33-45.

algorithms eventually flourished, unstable words morphed and twisted over centuries of distorted renderings across languages, religions, commercial and philosophical settings, texts and traditions, lands and seas.[28] Algorithms were strange and suggestive methods with a strange and suggestive name, prone to produce misinterpretation alongside insight. With algebra, another etymologically troublesome term traceable to a Medieval Arabic expression for repair from the same mythologized source,[29] algorithms have long mystified and misdirected while promising clear and easy resolutions.

Summing and Casting

In Robert Recorde's 1543 English vernacular numerical primer, *The Ground of Artes,* algorithm and arithmetic are still uneasy synonyms of unstable substance and unaccountable promise.[30] The book reigned as the preeminent vernacular English arithmetical primer for more than a century and a half, printed in sundry editions through the end of the seventeenth century. Attempting to make sense of elementary arithmetic at a time when trust in its reliable operation could not be presumed, the text gives a provocatively explicit account of error and repair in arithmetic.

Published with the comparatively new representational technology of the portable vernacular printed book, Recorde's dialogue introduces the fundaments of figurate arithmetic through a sequence of demonstrations and recapitulations between a master and a student.[31] The text repeatedly insists and demonstrates that algorithmic-arithmetical knowledge and practice are grounded in misprision, correction, and the wisdom that comes with long and attentive experience of subtle twists, exceptions, and accommodations.[32] A student who reduces algorithms to rote procedures has, for Recorde, missed the essential lesson that "rules without practise, is but a lyght knowledge" (14^v). Neither yet code nor craft, algorithms surpass light knowledge by joining contextualized rules to assiduous practice. The joint is repair, and the mechanism is remediation.

The book's lesson on Addition begins with the simple sum of 160 Latin books and 136 Greek ones, the three digits of the one number easily aligned with the three of the other

[28] See Liesbeth De Mol and Maarten Bullynck, "What's in a name? Origins, transpositions and transformations of the triptych Algorithm—Code—Program," in Janet Abbate and Stephanie Dick, eds., *Abstractions and Embodiments: New Histories of Computing and Society* (Johns Hopkins, forthcoming).
[29] Tatarchenko, "Algorithm's Cradle," this volume; Rashed, *Al-Khwarizmi: The Beginnings of Algebra* (SAQI, 2009).
[30] Robert Recorde, *The grou[n]d of artes teachyng the worke and practise of arithmetike, moch necessary for all states of men* (R. Wolfe, 1543), STC (2nd ed.) 20797.5, accessed on Early English Books Online.
[31] On literacy and numeral practice in English vernacular arithmetic, see Jessica Otis, "'Set Them To the Cyphering Schoole': Reading, Writing and Arithmetical Education, circa 1540-1700," *Journal of British Studies* 56 (2017): 453-482. See also John Denniss, "Learning arithmetic: textbooks and their users in England 1500-1900," in *Oxford Handbook of the History of Mathematics*, ed. Eleanor Robson and Jacqueline Stedall (Oxford, 2009), 448–467. On vulgarization and vernacular mathematics in a more recent context, see Jones, this volume.
[32] Barany, "'Some call it Arsmetrike, and some Awgryme': Misprision and precision in algorithmic thinking and learning in 1543 and beyond," in Ames and Mazzotti, eds., *Algorithmic Modernity* (Oxford, forthcoming). See also Reigeluth, this volume.

and added place by place without carries.[33] The student perceives the pattern and proposes next to add 848 sheep to 186 other beasts. After lining up the figures the student follows the rote procedure of partial summing from before, starting on the right: 8 and 6 make 14, which the student writes below the line. "Not so," interjects the master, for two reasons. First, the scholar erred by "goyng about to adde together ii. summes of sondry thynges" producing "a summe nother of shepe, nor other beastes, but a confused summe of both" (18^r). Having re-mediated hypothetical livestock into figures, the student must remediate the category error of improper admixture that this translation allowed. Categorical repair—itself effectuated through a translation between forms and genres of dialogue, imagined handwriting, mental summing, and print publication—becomes a lesson in the limits of arithmetical formalism.

Second, the student's error of mixing sheep and other beasts paralleled an error of mixing two-digit numbers and one-digit numbers. Instead, the master explains how the student should "kepe the article [the higher-place-value digit] in your mynde" to be carried (18^v). Place-value and carriage difficulties escalate further with the sum of armies of 106800 and 9400 soldiers, but really spin out of the comfortable collaboration of head and hand with the next sum, a mass of thirteen three- and four-digit numbers overflowing with nines and totaling 29057. "This," the student astutely declares, "semeth somewhat harde, by the reason of so many nombres together" (23^r). The student asks for reassurance that with practice such sums will become easier, and the master supplies this reassurance in part. "But," the master adds, "bycause of such greate summes, in whiche there may chaunce to be some errour" the proof of repeated practice must be supplemented with another kind of proof (Recorde uses the verb "prove" in both instances), replacing the complex sprawling sum with something less prone to errant reckoning.

This latter proof is the procedure of casting out nines.[34] The procedure in Recorde's primer develops the method as a series of manipulations and shifts in perspective, translating and transmuting numbers based on the properties of the numerals that represented them. First, disregard those problematic place values altogether and treat each digit as a unit. Next, add the units together, subtracting nine whenever the sum exceeds that greatest of numerals. Write the result "in some voyde place, by the ende of a lyne, for the better remembraunce" (23^v). Repeat this for the result of the original sum and write the digitally decimated outcome at the other end of the line in the void. If the two ends of the mnemonic line are alike, the sum is well done; if they differ, it is wrong. Eventually, in Recorde's discussion of division, the master distinguishes between the "co[m]men profe" of casting nines and checking more

[33] On carrying as a characteristic challenge in the history of arithmetic, algorithms, and mechanical computing, see Matthew L. Jones, *Reckoning with Matter,* op. cit. On algorithmic demonstrations' reliance on prepared and controlled settings that avoid characteristic limitations, see Elzway, this volume.

[34] In modern number-theoretic terms, this amounts to performing the sum again modulo 9. Whether or not you have encountered modular arithmetic, you may have learned a trick for testing divisibility by 9 that rests of the same principle: add up the digits of a number (repeating if the result has multiple digits) and if the end result is nine then the original number was divisible by nine, e.g. 453 becomes 4+5+3 = 12 which becomes 1+2 = 3, so 453 is not divisible by nine, while 468 becomes 4+6+8 = 18 which becomes 1+8 = 9, so 468 is divisible by nine. The principle is based on the fact that every power of ten has a remainder of one when divided by nine, so arithmetic equations that are true for base-ten numbers are also true if you treat the digits as separate numbers added together.

certainly by reversing the calculation in full (68^{v}), qualifying the simple right-wrong distinction from the test of casting nines in addition.[35]

As the sums become even more complicated, for instance mixing pounds and shillings and pennies, the procedure for casting nines grows more complicated apace. So complicated, in fact, that the student complains "I do not se[e] the reason of this" (33^{v}). Here, as repeatedly throughout the book, Recorde has the master counter not with a reason but with a rationale for deferring that lesson: "for harde it is for to occupye a yonge learned wytte w[ith] both the arte and the reasons of it all at ones" (34^{r}). The scholar eventually plays along, for instance asserting that "I understande you beste, when you do not stande longe in tellying the rule before examples, but propose some example, & then in declaryng it, brynge in the rules with all" (56^{r}).

When rematerialization gets hard, the solution is more rematerialization. The master asks the student to recall a carpenter measuring a board, pricking it and then drawing a number of dashes from the prick to help locate it, sometimes forming a cross to mark the spot.[36] Just as the carpenter's cross helps to keep track of a measured point, a cross on the page can help the student track and match the remainders from casting nines. The process of casting nines thus becomes a series of translations between the original sum and a mnemonic line or series of crossed lines, which Recorde later deploys for checking other operations after addition. Other mnemonic forms include a numeral table for the multiplication of digits (49^{v}), a "checker table" for combining these digital products (55^{v}), a new arrangement of figures (some crossed-out) for division (62^{v}), a cascading table for exponential progression (99^{v}), a "cro[o]ked draught of lynes" for the "Golden rule" of proportions (102^{r}), and print representations of manipulable counters on a counting table (117^{r-v}).

These translations and remediations appear here explicitly or implicitly as pedagogical and practical stand-ins for reason. Recorde's dialogue explicitly raises and then suppresses the question of why calculation techniques and tests work in order to make a point about reason, experience, expertise, and error. As with more recent technologies of mediated reasoning and their associated moral and epistemic economies, reckoning and casting involved a balance of rote technique, experience, contextual application, and compromise between the goals of computation and the limits of comprehension.[37] Sociotechnical histories of algorithmic implementation[38] can indicate the materially delimited values and compromises, the ideals and repairs, that license algorithmically produced knowledge from routinized transformations.

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[35] See Barany, "Awgryme," op. cit., on 10.
[36] On "pricks" and "points" (not quite epistemically identical) in early modern geometry, including Recorde's, see Barany, "'That small and unsensible shape': Visual representations of the Euclidean point in sixteenth-century print," *Spontaneous Generations* 6 (2012): 148-159.
[37] See Stephanie Dick, "AfterMath: The Work of Proof in the Age of Human-Machine Collaboration," *Isis* 102 (2011): 494-505.
[38] See Dick, this volume.

The Promise of Mediation

The media, contexts, production, and nature of mathematical knowledge changed dramatically in the first half of the nineteenth century. Significant improvements to the speed and cost of paper production facilitated new opportunities for paper-based mathematical exposition, circulation, and learning. For the first time in mathematical history, paper was abundant and even disposable for many advanced mathematics learners and researchers, facilitating new kinds of paper-based examination, study, and research practice.[39] At almost the same time, mathematicians, engineers, and scientific counterparts took up with zeal a pedagogical technology that had for centuries been primarily a medium of choral training, the large-scale blackboard.[40] Together, bulk paper and blackboards in settings of lecture and drill reconfigured the material conditions of writing and rewriting in the production of formalized knowledge. Changing how students and researchers produced and moved inscriptions between various situated and circulable media, how they transformed and remedied mathematical representations, altered in turn how they produced and engaged ideal worlds of theory.

The oldest records of blackboards in the École Polytechnique, the Paris military engineering institution at the front of the move to blackboard pedagogy in the nineteenth century mathematical sciences, are records of repair.[41] An 1805 note reflects the institution's struggles to develop an appropriate varnish for their new infrastructures of inscription. Blackboards could not be taken for granted as substrates for legible, functional, erasable chalk writing, and making them so was considered an appropriate concern for the elite chemistry professors in the École Polytechnique's employ. Repair and repainting recur at intervals in administrative record books in the school's early years, as do regular bulk purchases of chalk, before board maintenance and writing implements respectively fade into routine aggregated accounts of furniture and supplies. As blackboards became routinized features of the École Polytechnique's built environment, with repair and maintenance no longer remarkable as a component of institutional operations, they could be naturalized in the pedagogical routines and imaginaries of the school's budding and seasoned engineers.[42]

Today's blackboards are predominantly industrial ceramic products, which require cleaning but do not visibly degrade nor require regular painting in the way that most blackboards for most of the medium's material history did. In that longer history, repair was a central theme for both routine practice and commercial and pedagogical competition and

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[39] Andrew Warwick, "A Mathematical World on Paper: Written Examinations in Early 19th Century Cambridge," *Studies in the History and Philosophy of Modern Physics* 29 (1998): 295-319; Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago, 2003), ch. 3.

[40] Jessie Ann Owens, *Composers at work: the craft of musical composition, 1450-1600* (Oxford, 1997), 74-107; Barany, "Histories of mathematical practice: reconstruction, genealogy, and the unruly pasts of ruly knowledge," *ZDM Mathematics Education* 52 (2020): 1075–1086, on 1078-1080.

[41] These early records are discussed in Barany, "Unruly pasts," op. cit., on 1079-1080.

[42] In this spirit, remediation may offer an alternative or counterpart to the recent emphasis on maintenance in the historiography of infrastructure; on which, consider Andrew Russell and Lee Vinsel, 2016, "Hail the Maintainers," *Aeon,* online, 7 April, https://aeon.co/essays/innovation-is-overvalued-maintenance-often-matters-more.

innovation.[43] While slate was often usable for small-scale writing surfaces, before the industrial ceramic era most large-scale boards were built from materials that required some kind of paint or varnish. Blackboard painting, including its regular maintenance, sustained not just writing surfaces but institutions, and visitors to new settings remarked on blackboards' observable condition as evidence of institutions' resources and priorities.

Naturalists, geographers, surgeons, and many others took to blackboards for their dynamic representational possibilities over the nineteenth century, including depiction and illustration as well as formalization and abstraction. Blackboards in elite scientific settings, public expositions, and new systems of mass primary and secondary education became common elements that crossed social and scholarly contexts, connecting knowers and places of knowing through cognate experiences of mediation.[44] From drilling and examining military officer candidates to orienting and disciplining school children, blackboards were settings for representing, training, transforming, and repairing the postures, practices, and virtues of people: citizens, scientists, workers, participants in worlds of shared and place-based inscription, of seeing and knowing and repairing together.[45] Cultures and practices of disciplining people returned in the nineteenth century to new forms of theory-making based in recursive practices of analyzing and repairing errors and imprecisions in experiments and ideals.[46] By the latter part of the century, blackboards were familiar stages for producing and examining theories, and their abundance had a deracinating effect on the theories written on them. Repeated remediation, writing and revising and rewriting and revisiting across naturalized infrastructures of representation, made particular settings of theory making appear equivalent and universal.

The material proliferation of blackboards made them an especially recognizable icon for the material conditions of sublimating abstract thought. So recognizable, in fact, that this iconic character extended to many settings where the dynamic material inscription blackboards enabled was primarily decorative. The "scientist at a blackboard" genre of portraiture may be the most obvious twentieth-century example of the blackboard as stand-in for the power and reach of abstract thought, notwithstanding the functional illegibility of much of blackboard theory-making outside of its immediate settings of inscription[47] and the (perhaps consequent) tendency for blackboard portraiture to be staged, with writing (where present) made for purposes of portraiture rather than for situated reasoning.[48] Literary critic Roland Barthes famously contrasted the messy blackboards of Einstein photographs with the clean sublimation of an inspired formula in cartoons of the iconic genius physicist, but even this contrast reflects Barthes's willingness to read into the photographic record a "visible complexity" that differs substantially from the strikingly posed and premeditated

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[43] Ackerberg-Hastings, Kidwell, and Roberts, *Tools of American mathematics teaching, 1800–2000* (Johns Hopkins, 2008), 21-34.
[44] Barany and Cowles, "Thirteen Ways."
[45] Barany and Cowles, ibid. See also Christopher Phillips, "An officer and a scholar: Nineteenth-century West Point and the invention of the blackboard." *History of Education Quarterly* 55 (2015), 82–108.
[46] Kathryn M. Olesko, *Physics as a Calling: Discipline and Practice in the Königsberg Seminar for Physics* (Cornell, 1991), ch. 10.
[47] See Barany and MacKenzie, "Chalk."
[48] See Barany and Cowles, "Thirteen Ways."

performance of blackboard inscription evident in extant portraits.[49] In a recent collection of portraits subtitled "Mathematicians and their Chalkboards," the mathematicians themselves are entirely absent from the frame, with the inscribed chalkboard alone representing the dynamic inspiration of situated thought.[50]

GLOBALIZING UNIVERSALS

The converse to the blackboard in the material reconfiguration of modern mathematical sciences was the printed article, the written-up and circulable form of the formal knowledge produced by reading down with chalk. By the end of the nineteenth century, an internationalist mathematical elite expressed their hope and anxiety over a burgeoning world of research in print through a preoccupation with the problems and possibilities of mathematical bibliography.[51] Identified at the opening session of the first International Congress of Mathematicians in 1897 as the "searing question" that most urgently motivated international cooperation,[52] the ambition to sustain a rapid and comprehensive infrastructure of mathematical bibliography responded to, reinforced, and reconfigured a nascent sense from the mathematical periodical culture of the late nineteenth century that there was such a thing as a single worldwide literature of mathematics and that it was incumbent upon mathematicians to keep up.[53] Mathematicians through the early twentieth century understood this literature mostly retrospectively, as a dimension of library collection and research practices that complemented the limited updates and questions that could be shared in closer to real time as correspondence.

Well into the twentieth century, this sense of the literature as a unified coherent body of knowledge to be followed as it unfolded could not be presumed in all corners of the mathematical world. Especially in traditional centers of research with sufficiently large and active research communities, national and even local literatures together with lectures and other in-person sources of mathematics offered sufficiently wide horizons for most. The profound and typically under-credited difficulty of producing mutual understanding in mathematics, a difficulty still greater at a distance, meant there were many barriers and comparatively few incentives to imagining a world of interlocutors any wider than necessary for the essential social, intellectual, and material sustenance of a local mathematical career. Pressure to unify the mathematics literature and develop the apparatus to do so came from those mathematicians close enough to the center to have robust access to recent publications

[49] Roland Barthes, *Mythologies,* trans. Annette Lavers (London: Paladin, 1972 [1957]), 68–70, see Barany and Cowles, "Thirteen Ways."
[50] Jessica Wynne, *Do Not Erase: Mathematicians and their Chalkboards* (Princeton, 2021).
[51] Michael J. Barany, 2020, "Abstract Relations: Bibliography and the Infra-Structures of Modern Mathematics," *Synthese.* See also Csiszar, this volume.
[52] Rudio, F. "Über die Aufgaben und die Organisation internationaler mathematischer Kongresse," in F. Rudio (Ed.), *Verhandlungen des ersten internationalen Mathematiker-Kongresses in Zürich vom 9. bis 11. August 1897* (Teubner, 1898), 31-42.
[53] Michael J. Barany, "Placing a Global Mathematical Literature," in *CSHPM Annals,* ed. Maria Zack and David Waszek (Birkhäuser, forthcoming).

but far enough to the periphery to depend on those publications to feel a part of sufficiently engaging programs of research.[54]

A decisive shift arrived in the 1930s with the launch of the German publisher Springer's *Zentralblatt für Mathematik,* an abstracting journal aiming to be as international as possible in scope and as rapid as possible in production. The journal's catalogue of citations and abstracts helped the publisher advertise current literature by transmuting full-length publications into a format that could be much more efficiently distributed and digested. Though the *Zentralblatt* was neither global nor instantaneous, it suggested the possibility of the kind of global real-time literature for which mathematicians called in 1897. The *Zentralblatt* was, moreover, sufficiently broad and rapid that mathematicians could begin to use it both as an index to a stable library of settled knowledge and as a guide to current research with emerging methods and questions that might be incorporated into one's own research programs. Accordingly, the mathematical abstract format mobilized precisely the kinds of relational and contextual information that could most effectively bridge research activity in distant locations.[55]

To span the gap between what appears in a review, either in the *Zentralblatt* or in its American counterpart *Mathematical Reviews* (launched with urgency in 1940 in response to a precipitously deteriorating situation for international science in Germany[56]), and what is necessary to engage substantively with the research reported therein, mathematicians engaged a variety of mediated methods of repair. Extensive and sometimes contested systems of classification enrolled work in the forms of citations and abstracts into specific methodological and conceptual traditions and social and institutional frameworks.[57] By actively reviewing related articles, a small community of mathematicians could produce the appearance of a much larger, more coherent, and more exciting field of research by exploiting the review format to reframe others' work in terms of the reviewers' research program.[58] Mathematical abstracting as a distributed global practice thus supported a pace and scale of connection within conceptual programs that far outstripped what mathematicians could achieve by sharing articles piece by piece or subscription by subscription. Remediating their results into the limited but circulable format of the review let mathematicians remotely sustain a sense of connection and mutual relevance.

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[54] See Evan Hepler-Smith, 2016, *Nominally Rational,* (PhD diss., Princeton) on the center-periphery dynamics of the Strasbourg chemists who led late-nineteenth century efforts to universalize and integrate chemical information.

[55] Barany, "Wordplay" (cit. n. 17); Barany, "Abstract Relations" (cit. n. 52).

[56] Reinhard Siegmund-Schultze, "'Scientific control' in mathematical reviewing and German-U.S.-American relations between the two world wars," *Historia Mathematica,* 21 (1994): 306–329.

[57] Barany, "Abstract Relations"; François Lê and Anne-Sandrine Paumier, "De la science comme classification à la classification comme pratique scientifique: quelques réflexions à partir de deux cas mathématiques," *Cahiers François Viète* ser. III, 1 (2016): 9-34. On the politics of classification, see also Drabinski, Emily. "Queering the Catalog: Queer Theory and the Politics of Correction." *The Library Quarterly: Information, Community, Policy* 83 (2013): 94-111; Geoffrey C. Bowker and Susan Leigh Star, *Sorting Things Out: Classification and Its Consequences* (MIT, 1999).

[58] Barany, "Wordplay"; Anne-Sandrine Paumier, *Laurent Schwartz (1915–2002) et la vie collective des mathématiques* (PhD dissertation, Université Pierre et Marie Curie, 2014), §2.3.

Ultimately, however, durable collaborations and communities required travel. Such communities spread over long distances required travel that could be prohibitively costly. Starting in the 1920s, new infrastructures of funding and evaluation opened possibilities for many more mathematicians to travel far farther than their counterparts of prior generations. These infrastructures in turn hinged on mediated practices of translation and repair.[59] With maps, surveys, circular letters, postcards, index cards, file systems, and other means, mathematicians and their sponsors formed, navigated, and rectified hybrid and provisional views that channeled resources to those who could be made to fit the right institutional templates. Rematerializing signifiers of theories, people, and communities was a means of imagining a far more connected and mutually intelligible mathematical world than was routinely manifested in the day-to-day practices of globally-oriented mathematicians. Institutional media consolidated and remedied the partial and limited connections and formations of professional mathematical interaction, allowing mathematicians to sustain institutions and disciplinary networks at transnational scales.

REMATERIALIZING 1+1

As mathematicians pushed the limits of complex theories and large-scale communities, they returned again and again to the fundamental and emphatically evident basic facts of arithmetic.[60] One canonical version of the story of this persistent arithmetical preoccupation starts with the nineteenth-century advent of set-theoretic foundations of arithmetic as an outgrowth of philosophical and technical debates about proof and determinacy in mathematical analysis, and extends to the crises of undecidability and incompleteness associated with Kurt Gödel in the 1930s.[61] Successive philosophers and mathematicians constructed elaborate mechanisms of logical stipulation and calculation that bore little resemblance to how most people intuit and reckon, but that permitted new kinds of analysis and decision. An essential feature of these new foundations was their overwhelming dependence on in-principle projection to coherent wholes. They propounded rules and procedures that appeared absolutely rigorous at the scale of individual deductions at the cost that anything recognizable as a meaningful conclusion outside of narrow philosophical

[59] Michael J. Barany, "The Officer's Three Names: the formal, familiar, and bureaucratic in the transnational history of scientific fellowships," in Krige (ed.), *How Knowledge Moves: Writing the Transnational History of Science and Technology* (Chicago, 2019), 254-280; Barany, "Rockefeller Bureaucracy and Circumknowing Science in the Mid-Twentieth Century," *International Journal for History Culture and Modernity* 7 (2019): 779-796; Barany, "Fellow Travelers and Traveling Fellows: The intercontinental shaping of modern mathematics in mid-twentieth century Latin America." *Historical Studies in the Natural Sciences* 46 (2016): 669-709; Barany, "Organizational Practice," op. cit.

[60] The empirical evidence of the basic facts of arithmetic—together with the idea that certain basic facts are or are not self-evident—has its own long history that comes in and out of contact with the history of logical foundations. From Hume and Mill (e.g. the latter's *System of Logic,* 1843, 139-145) to Lakatos (*Proofs and Refutations,* op. cit.) and Bloor, (*Knowledge and Social Imagery,* op. cit.), numbers and basic arithmetic have been canonical "base cases" or "hard cases" for philosophies of cognition and evidence; see also Michael J. Barany, "Of Polyhedra and Pyjamas: Platonism and induction in meaning-finitist mathematics," in de Freitas, Sinclair, and Coles (eds.), *What is a mathematical concept?* (Cambridge, 2017), 19-35. These have often been subtly or unsubtly connected to ethnological projects, including those of scientific racism; see Barany, "Savage Numbers," op. cit.

[61] Grattan-Guinness, *Mathematical Roots,* op. cit.

concerns would require so many individual deductions as to be unfollowable and even unrecognizable.

Two famous demonstrations of the preposterous lengths these reformers were prepared to go to trade tractability for formal rigor are Bertrand Russell and Alfred North Whitehead's turn-of-the-century proof of the proposition 1+1=2 and Nicolas Bourbaki's midcentury definition of the number 1. The goals of these respective foundational projects were, of course, rather different and more capacious than establishing the full, true, and universal meaning of these particular elements of arithmetic. In both projects, it was a social, moral, and philosophical decision not to start with rudimentary addition or the conventional unit of counting. Choosing other starting points and correspondingly pushing 1+1 or 1 itself as far into the body of the philosophical system as possible was a narrative choice that did little to change the everyday meanings of 1+1 or 1 but did everything to announce the ambition and character of the philosophical project. The logic of Russell and Whitehead's *Principia* and Bourbaki's *Elements* was so profound, it would seem, that the usual starting points of arithmetic emerge only down the line as relatively insignificant corollaries to staggeringly immense edifices of stipulation and deduction.

Russell and Whitehead's (1910-1913) *Principia* begins (vol. 1, 1) by announcing three goals: re-founding mathematics on as few and as fundamental "primitive ideas and primitive propositions" as possible; expressing the resulting mathematics "in the simplest and most convenient notation possible"; and resolving paradoxes in symbolic logic and set theory that had gained attention in the preceding decades. Tellingly, the authors immediately observe that the first and third purposes "often compel us to adopt methods, definitions, and notations" at odds with the second. Both representation and cognition required repair, but their respective remedies could seem to be in conflict with each other. Russell and Whitehead's methodological rationale came in part from a deep suspicion of ordinary language, which they declared an intention to discard in favor of grammatically and semantically unambiguous symbols whose "very abstract simplicity … defeats language" (2). Ordinary language, they asserted, expresses complex ideas simply but dissolves "to an intolerable prolixity" when addressed to simple matters like the expression "one is a number" (ibid.). Where the authors saw symbolic reasoning to be in contrast to the work of imagination in mathematical thought (3), it was precisely a surplus of unaccounted imagination that makes it possible to credit hundreds of pages of symbolic deduction as a meaningful translation of facts like 1+1=2. This last claim appears on page 86 of volume 2 of the *Principia,* more than seven hundred pages after Russell and Whitehead's warning about the intolerable prolixity of explicating the simple.

Nicolas Bourbaki's *Elements of Mathematic* was also motivated by a concern for systematic rigor in mathematics and a skeptical view of ordinary language. For the French mathematicians behind the iconic pseudonym,[62] however, the goal was less to resolve paradoxes in the philosophy of language and representation than to give a universal basis for

[62] See Michael J. Barany, "Impersonation and personification in mid-twentieth century mathematics," *History of Science* 58 (2020): 417-436.

reasoning abstractly to unify and integrate modern mathematics.[63] As working mathematicians, Bourbaki's collaborators (as the authors behind the pseudonym called themselves) valued the fruitfulness of informal language and reasoning in creative mathematics. To link this to a foundation of symbolic formal rigor, Bourbaki emphasized the importance of "experience and mathematical flair" as a substitute for "tedious" formal deductions.[64] So-called "abuses of language" would let a witting mathematician reason using ordinary terms while keeping attention on the symbolic rigor that connects that reasoning in principle to Bourbaki's unified mathematical foundation.[65] Bourbaki's number 1, as distinct from the convenient symbol "1" and the ordinary word "one," is the subject of a condensed four-line definition in chapter 3 of Bourbaki's *Set Theory*. These four lines, Bourbaki asserts, abbreviate a full symbolic definition that would run to "dozens of thousands of signs"—an underestimate by some seven orders of magnitude according to a later calculation.[66] Evidently, what mattered was the fact of "1" being represented by an incomprehensibly large amalgam of symbols shortened to a still challenging dense symbolic abbreviation, not a rigorous accounting of just how incomprehensibly large that amalgam would be if pursued in its full formal tedium.

Neither Russell and Whitehead's nor Bourbaki's definitions should be read at face value, and both works signal that these representations should be understood to some extent as tongue-in-cheek. Russell and Whitehead's proof that 1+1=2 is followed immediately by the claim "The above proposition is occasionally useful" and a clearly deadpan list of "at least three" uses in subsequent propositions of the *Principia*. Bourbaki's excess of circumlocution and flippant (and massively underweight) estimate of their formulation's full heft is a winking performance of over-extensively defining something that needs no definition, so as to frame a mathematical system where such definitions could and did make a difference.[67] The collaborators of Bourbaki, in particular, used these kinds of winking translations and wordplay to build and maintain social and scholarly communities in the new geopolitical and intellectual environments of the mid-twentieth century.[68] Substitutions, shorthands, translations, and remediations let always-provisional social and conceptual approximations stand in for in-principle worlds of people and theory that always necessarily vastly exceeded what was accessible, comprehensible, or meaningful in a given time or place.

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[63] Liliane Beaulieu, "A Parisian Café and Ten Proto-Bourbaki Meetings (1934-1935)," *The Mathematical Intelligencer* 15 (1993): 27-35; Leo Corry, "Writing the ultimate mathematical textbook: Nicolas Bourbaki's *Élements de mathématique*," in Robson and Stedall, *Oxford Handbook*, op. cit., 565-588.
[64] Bourbaki, *Éléments de Mathématique, Livre I: Théorie des Ensembles* (Hermann, 1954), 1-2.
[65] Barany, "Wordplay," esp. 276-278.
[66] A.R.D. Mathias, 2002, "A Term of Length 4 523 659 424 929," *Synthese* 133(1/2): 75-86.
[67] The extent of this difference has, however, been questioned. David Aubin, "The Withering Immortality of Nicolas Bourbaki: A Cultural Connector at the Confluence of Mathematics, Structuralism, and the Oulipo in France," *Science in Context* 10 (1997): 297-342; Corry, *Modern Algebra and the Rise of Mathematical Structures* (Birkhäuser, 2004).
[68] Michael J. Barany, "'A Young Man's Game': Youth, gender, play, and power in the personae of mid-twentieth century global mathematics" in Kirsti Niskanen and Michael Barany, eds., *Gender, Embodiment, and the History of the Scholarly Persona: Incarnations and Contestations* (Palgrave, 2021), 21-53; Liliane Beaulieu, "Bourbaki's Art of Memory," *Osiris* 14 (1999): 219-251; Beaulieu, "Jeux d'esprit et jeux de mémoire chez N. Bourbaki," in Abir-Am (ed.), *La mise en mémoire de la science: pour une ethnographie historique des rites commémoratifs* (Editions des archives contemporaines, 1998), 75-123.

CONCLUSION: CASTING BETWEEN CODE AND CRAFT

The elaborate formalization of 1+1 or 1 here functions as a kind of social and symbolic casting, analogous to the casting that bridged tractable examples and complex figurations of potential arithmetic in Recorde's *Ground of Artes.* In early modern and high modern remediations of basic arithmetic alike, situated mechanisms of mediated reasoning supported fantasies of universal method, theory, and community that licensed grand claims about the power and certainty of formal and symbolic methods imagined to transcend provisional and faulty lived experiences of practice. Situated algorithmic craft and programmatic algorithmic code are not so much two separate dimensions of the practice and theory of algorithmic knowledge as they are indices of the interplay of accommodation and correction that lets algorithmic thinkers and doers operate simultaneously in the messy realities of errors, bugs, and miscalculations and the ideal systems of universal, formal, deterministic reason. The craft-work of error and correction licenses and disciplines the code-work of the fantasy of coherent and exact algorithms functioning according to plan and intention, and this code-work simultaneously licenses and disciplines the craft-work.

In 2009, a ragtag ensemble of professional and amateur mathematicians came together in the comments sections of the blogs of two distinguished mathematicians—one in Cambridge, England, and one in Los Angeles, California—with the goal of proving a theorem in combinatorics. They imagined their effort as an experiment in mathematical collaboration, using new possibilities for dynamic long-distance mediated public reasoning to foment a kind of crowd-based creativity. They quickly found that they were not, as one organizer put it "free to make up conventions as we go along," and a tremendous share of their collaborative effort involved testing and accommodating the limitations of their media of blogs and eventually wikis and other online platforms and modalities.[69] They erred and corrected, aspired and adapted. They found that much of what they imagined they could do together still ended up happening offline, but that their online community could indeed do some things (particularly regarding the management of information and partial results) better than any one of them could conveniently do alone. They imagined a seamless in-principle collectivity of reasoning; they tacked between online and offline media, between translating and transforming and remedying, between code and craft. They proved the theorem.

In 2012, a distinguished mathematician in Kyoto uploaded the first full version of a long and difficult series of articles including a new theory and a claimed proof of a major open conjecture.[70] As is now typical for mathematical claims of such recognized importance and complexity, these prompted extensive discussion among the small community of mathematicians prepared in principle to understand the result, as well as widespread

[69] Barany, "'[B]ut this is blog maths and we're free to make up conventions as we go along': Polymath1 and the Modalities of 'Massively Collaborative Mathematics.'" *Proceedings of the 6th International Symposium on Wikis and Open Collaboration,* Gdansk, Poland, 2010 (ACM, 2010).

[70] An insightful discussion of the events up to 2020 is in Colin Rittberg, "Intellectual humility in mathematics," *Synthese* (2021). A major forum for skeptical mathematical discussion has been the blog of mathematician Peter Woit, e.g. "Latest on abc," 3 April 2020, https://www.math.columbia.edu/~woit/wordpress/?p=11709; "ABC is Still a Conjecture," 4 March 2021, https://www.math.columbia.edu/~woit/wordpress/?p=12220.

speculation among amateur and professional mathematicians at large. These discussions, also typically, raised questions about ambiguities or missing elements of the proposed result, prompting revisions and further attempts to explain and repair the argument. The discussions have ranged over a striking variety of media and rematerializations, including formal and informal internet documents, animated videos, live and recorded blackboard and whiteboard presentations, blog discussion threads, published journal articles and reviews, and personal conversations. Rather than build toward consensus, debate over the result is now at an apparent impasse including sometimes-acrimonious exchanges between mathematicians convinced of the result's correctness and those who believe they have identified a decisive error. After the Kyoto mathematician drew criticism for his reticence to travel to try to convince foreign experts of his result in person, two skeptics in 2018 travelled to Kyoto for a week and came away reinforced in their skepticism, one later citing his willingness to travel in a *Zentralblatt* review affirming his doubts about the mathematical claim.[71]

The self-styled "Polymath" collaboration, inspired among other sources by a particular image of radical collectivity associated with Bourbaki (in homage to whom the Polymath collaborators ultimately published their result under a collective pseudonym), is a recent example of a long-running mathematical tradition of producing, navigating, and imagining beyond distributed patchworks of partial mutual understanding through various and provisional acts of sociomaterial coordination.[72] The contested Kyoto result is a recent example of long-running tensions and limits of that mathematical tradition, where sociomaterial reproduction and remediation fails to sustain distributed consensus in purported universal knowledge. Centering the work of remediation in this history makes clear the mutual constitution of situated repair and disciplined fantasy behind the practice and possibility of formal reasoning.

[71] Peter Scholze and Jakob Stix, "Why abc is still a conjecture," https://ncatlab.org/nlab/files/why_abc_is_still_a_conjecture.pdf. Scholze zbMATH review of Mochizuki, Zbl 1465.14002, 2021.
[72] Stephanie Dick, this volume; Michael Harris, *Mathematics Without Apologies: Portrait of a Problematic Vocation* (Princeton, 2015); MacKenzie, *Mechanizing Proof,* op. cit. ; Alma Steingart, "A group theory of group theory," *Social Studies of Science* 42 (2012): 185-213.