Inverting Hierarchies: The sociology of mathematical practice

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Abstract

Sociology originated in the mid-nineteenth century from a new confidence in the power of science to explain the world on a mathematical foundation. Both mathematics and sociology transformed over the ensuing century, inverting the hierarchical relationship from sociology as a mathematics-based science of complex human configurations to mathematics as a complex science based on social institutions. That is, where sociology began as the hard case for mathematics, it became possible to see mathematics as the hard case for sociology. In this light, we examine a number of motivations and provocations for the sociology of mathematics, from language and social cognition to Marxist and materialist skepticism of ideology, and sample the evidence and arguments brought to bear on the sociology of mathematics at a range of scales.

Keywords: mathematics, social studies of mathematics, sociology, sociology of mathematical practice, sociology of mathematics, sociology of scientific knowledge (SSK)

Introduction: Mathematical and Social Order

Mathematics and society are based upon two radically different but essentially identical forms of order. Where mathematics is made up of *abstract* entities and *formal* structures governed by *universal* laws of reasoning, society counts *human* entities and *social* structures governed by *particular* laws of relation. Both are made from methods and modes of intersubjective agreement, of human groups organizing themselves and their senses of reality around something that is ultimately in their heads, a shared imagination. The evidence of the working of mathematics and of society is in collective episodes of argument and validation, of proposition and correction, by which people make sense of the possibility of having something invisible, immaterial, and greater than themselves in common.

Mathematics is a very old science, arguably dating back thousands of years. Sociology is a very new one, aged just a couple of hundred. The former was integral to the origin of the latter. To give one necessarily fabular tale of beginning: sociology started as the opposite polestar to mathematics in the positive philosophy of Auguste Comte, whose ungainly coinage of *sociologie* urged the possibility of a systematic science of society built on a hierarchy of more fundamental physical and mathematical sciences with abstract mathematics at the very foundation (Comte 1842).

The ambition for a science of society, for a social physics (as Comte and many contemporaries also termed it), grew from early nineteenth century transformations in the relationship between mathematics and society. Propounded in new scales and uses of comprehensive training in calculus-based engineering (see Belhoste 2003), this transformation instilled a new confidence among certain

European elites in the engineerability, and accordingly the mathematical knowability, of the increasingly urbanized social world after the Industrial Revolution. Comte's sociology became one of many sometimes-conflicting mid-nineteenth-century sciences of society—mathematical statistics derived from perhaps an especially notable contemporary variation of 'social physics' (Hacking 1990)—each with its own social and scientific legacies and relationships to mathematics.

In this light, the sociology of mathematics has a kind of *ouroboros* character: it circles back, it eats its own tail. If the apparent universality of mathematical order underwrote the promise of a scientific account of the social world, such accounts can have the paradoxical effect of localizing and particularizing socially produced mathematics. Sociologists and mathematicians, alike and in their own ways, revel in paradoxes. They show the limits and the contradictions of what may be agreed, of the terms on which people may come together.

The next section reviews some of the major areas of sociology that have had something to say about mathematics. We then sketch how the sociology of mathematics, as such, has been constructed as one such limit, or contradiction, and develop some of the consequences of that view. Our analysis dwells at some length on the sociology of David Bloor, not necessarily because his work is representative or exemplary of the sociology of mathematics but because it has been a widely shared point of reference in the field, an excellent provocation, and a fruitful illustration of many of the themes discussed here. We identify a thread of interest in collective and societal responsibility in mathematics, which we follow from early Marxist analyses of the discipline through the proposals of Sal Restivo and others motivated by the socially critical potential of sociologizing mathematics. Finally, we consider the varieties of evidence at different scales that have informed such sociologies, and some of the conclusions that can be drawn from these, including understandings of small-scale practices, labor, communities, and large-scale networked knowledge and authority.

Sociologies of Knowledges

Ideas about the relationship between mathematical and social order are at least as old as the oldest societies that were large and complex enough to put mathematics to social purposes. As an apparatus of recording and accounting, mathematics began as an aid to memory and reconciliation, as far as the oldest surviving mathematical records appear to show. Mathematics stabilized valuable entities across time and among people. Its stabilizing uses made it useful in claims about justice, initially with measurements and resources and later in other domains of contestation and coordination, including (a very long time after the earliest mathematical records) those of civic and religious authority (Robson 2008, Desrosières 1998). These uses and expectations of mathematics constituted claims about its social conditions and implications.

Such claims can be identified in numerous traditions, many of very long standing, including traditions of philosophy as well as the practice of mathematics. They include invocations such as the famous heading to Plato's Academy, forbidding those ignorant of mathematics from entering. The idea of mathematics as a prerequisite to organized knowledge has never been unanimous or uncontested, but runs from before Plato's time through to our own times. After the trivium of grammar, logic, and rhetoric, the quadrivial mathematical subjects of arithmetic, geometry, music, and astronomy made up the foundation of the medieval liberal university curriculum. Vernacular arithmetician Robert Recorde claimed in 1543 that numbers were "the ground of all men's affairs" (see Barany 2023). The pedagogical status of mathematics supported broader claims about the relationship between the subject and the intrinsic characteristics of human thought and perception, notably significant to Kantian metaphysics among many philosophical traditions.

Sociologies of mathematics draw from these long traditions of reasoning about relationships of mathematics and society. These sociologies have come, in turn, from a variety of distinct and often mutually disconnected scholarly traditions concerned more explicitly and systematically with such society-knowledge (including society-mathematics) relationships. There has been no single or even dominant disciplinary or methodological home for social studies of mathematics, which have ranged across and often combined social scientific, anthropological, humanistic, philosophical, pedagogical, and other modes of inquiry.

For the purposes of this chapter, such an ungainly and unruly cluster of knowledge about knowledge is drawn together by a shared attention to the conditions and labor of producing and reproducing theoretical mathematics in the context of social institutions. That is, the sociology of mathematical knowledge is distinguished by seeking the interpersonal and socially organized dimensions of the most personal and logically-governed forms of knowing. This view of the field favors the elements that are closer to the mainstreams of the philosophy of mathematical practice, at the expense of social accounts of the uses of mathematics and quantification, including their social and sociological uses. These latter topics have at times crossed fruitfully with what might be called the sociology of 'mathematics itself' (notably, for instance, in the work of Donald MacKenzie, e.g. 2001), but have tended to distinguish between the socially manifested meanings and implications of mathematical reckoning and the situated production of those reckonings as such.

Institutional contexts of mathematics include formal organizations—educational institutions, research institutes, states, corporations, and so on—as well as the constellations of norms, expectations, and more or less formally organized ways of understanding and being in the world that mediate participation in social groups of various kinds connected to mathematical work (see Barany 2022; a summary of the main sociological uses of the term "institution" can be found in Miller 2019). Institutions can be understood at a range of scales, from how an individual experiences and navigates institutional forms to how such forms structure roles and interactions at the scale of an entire society or of interacting social groups. The sociology of mathematics has tended to focus on the smaller end of these scales, closer to the activities of intellection and communication of shared interest to the philosophy of mathematical practice. However, especially in sociological studies of methods and systems of education or on patterns of mathematical careers or funding, scales of analysis can grow and interpenetrate, linking the personal to much larger patterns of thought and action.

Sociologies of knowledge and of mathematics interpret institutions from a variety of methodologies and perspectives, reflecting these fields' decidedly mixed parentage from many different branches of sociology, anthropology, psychology, and other humanistic and scientific disciplines. Some generalize and aggregate, some particularize and individualize. Sociologies of mathematics, like philosophies of mathematics, may be unusual in the degree to which individual experiences of cognition or sense-making are taken as general or exemplary on the basis of the presumed generality of laws and principles of mathematical reasoning: many accounts of how mathematics works in general (sociologically) are based on very few social individuals or contexts.

The precise relationship of individual experiences of mathematics to the kinds of phenomena discernible on larger scales can make a great difference, analytically and methodologically. How one understands the nature of mathematical consensus and of collective reasoning changes what it means for individuals and communities to know and share mathematics. Producers and users of mathematical knowledge manage to agree on a lot, and they often see their agreement as being special in form, character, and even logical necessity. However, the social and historical fact of mathematics is that disagreement, misunderstanding, error, and variant comprehension are regular and routinized parts of mathematical research, learning, and communication. Sociological (particularly microsociological, ethnographic and microhistorical) approaches cast light on the process of achieving mathematical agreement before disagreements are folded into consensus and erased.

Constructivist sociologies see such variation as evidence that mathematical consensus is only ever provisional, and is made in social groups from a variety of ideas, experiences, practices, and values that combine to support particular senses of what one should accept and how one should reason. Among a variety of approaches, one may study such social constructions phenomenologically, examining how mathematics manifests in the lived experience of its users and producers, or one may study these ethnographically at the level of cultural forms and norms and group dynamics of sense-making. Constructivism is often opposed to realism or Platonism, two perspectives that prioritize claims that mathematics derives ultimately and inevitably from stable underlying truths that can be temporarily missed or misunderstood but that are not meaningfully revisable. This distinguishes sociology of mathematics from its object: while being a Platonist or anti-Platonist is typically understood to make no practical difference for a mathematician's daily work, these opposing epistemological perspectives result in entirely different approaches to sociological analyses of mathematics, the study of that daily work. Whether disagreement and negotiation are part of mathematical knowledge or extrinsic matters that circumscribe mathematical knowledge alters what needs to be explained about processes of knowledge making and sharing in mathematics, and how these can be explained.

The status of dissensus matters both for settled or "textbook" knowledge and for open, unresolved, dynamic, or contested mathematics in the making. Sociologies of mathematics often focus on the latter as a circumstance where assumptions and processes of reasoning are expected to be more explicit, as people try to develop their understandings and to convince each other. Settled mathematics can be approached with related methods by focusing on contexts of learning, mobilizing the fact that settled knowledge is only ever settled for those who know it, and there is a constant supply of learners who navigate the often unsteady passage from ignorance to acceptance and even comprehension of established knowledge.

Making a Hard Case

In Comte's hierarchical philosophy of knowledge that was one of sociology's nineteenth century points of origin, sociology was the hard case for mathematics, the pinnacle of a system of ever complexifying structure based on mathematics and verging toward comprehending everything. Over the twentieth century, this relationship inverted. A significant scholarly tradition came to construe mathematics as a hard case for sociology, perhaps the quintessential hard case for the sociology of knowledge.

Comte called his philosophy Positivism, referring to the combination of empirical facts and reasoning on which he claimed rested all positive knowledge, as opposed to lesser theological or metaphysical knowledge. In the early twentieth century, a critical confluence of philosophical interest in the linguistic foundations of both empirical description and logical reasoning gave rise to a different program in the philosophy of knowledge, also sometimes called positivism. Both positivisms were projects of unification and ordering, explaining how the modern sciences related to each other and to the nature of human thought.

Of their many differences, the most significant for understanding the inversion of mathematics and sociology as hard cases may be the respective positivisms' account of where mathematics comes from. For Comte, mathematics emerged as the first positive science from the problem of indirectly measuring magnitudes, based on the empirical and rational study of magnitudes' relations. For the later positivists, the key relations were between symbols, not magnitudes, and mathematics emerged as a semiotic or representational science built from the logical order and interpretation of written signifiers. This semiotic reorientation placed an emphasis on understanding, and on mathematics as an achievement of rational representation (if not necessarily communication, inasmuch as one comprehending mind sufficed).

When mathematics was a science of relations among worldly magnitudes, cognition and comprehension were importantly peripheral to the underlying stuff of the subject. As mathematics came to be located in symbolic representations, language and meaning became more central to characterizing the human production and understanding of mathematics, if not what it was exactly that was being symbolically understood. Language and meaning are social things. The scholarly possibility of sociology of mathematics (in its form as a sociology of knowledge) came from a refocusing of the epistemology of mathematics around individual and collective linguistic representation. The enduring notion of the underlying truth or reality of mathematics as existing outside of language, in worldly relations or in the basic logical forms of knowledge itself, meant that such sociologies of mathematics were founded in a paradoxical project of characterizing the social conditions of knowing something intrinsically independent of the social world.

Ludwig Wittgenstein has come to represent the centrifugal dynamic of mathematics as language in the first half of the twentieth century. It is common to divide his career into an 'early' and 'late' period. The former, defined by his immersion in the symbolic architectures and paradoxes of turn-of-the-century logicism and represented by his *Tractatus Logico-Philosophicus* (1922), epitomized logical mathematical knowledge as the ultimate product of an orderly propositional system. The latter, shaped by the tumult of interwar philosophy and represented in his *Philosophical Investigations* (1953) and *Remarks on the Foundations of Mathematics* (1956), emphasized the conventionality of linguistic understanding, including the linguistic understandings that underwrite knowledge and agreement in mathematics.

Wittgenstein was no sociologist and a very unconventional philosopher, but his provocative writings, mostly published posthumously, have influenced sociologists and nucleated major debates about the linguistic and cognitive bases of foundational knowledge in social contexts. The later Wittgenstein's emphasis on contextual and informal understanding and the provisional nature of mathematical agreement was inspirational to a postwar generation of scholars approaching mathematics from a multidisciplinary critical perspective that blended psychology, anthropology, social theory, and other fields into critical reconsiderations of the foundations of knowledge. In Wittgenstein's posthumous aphorisms, read through these approaches, the most banal mathematics—counting in a sequence, multiplying numbers—pulsed with indecision and shed their veneer of inevitability.

One of the most enthusiastic and controversial such re-readers of Wittgenstein was David Bloor, one of the formulators of the strong program in the sociology of knowledge, part of a cluster of constructivist sociologies that came to be known as the Edinburgh School of the sociology of science and technology. The strong program aimed to take a naturalistic view of knowledge, studying it empirically through the terms and manifestations observable wherever knowledge claims are made and collectively challenged or respected (Bloor 1976, 2-3). Regarding mathematics, the strong program contrasted with a 'weak program' of identifying a diversity of mathematical cultures and traditions, seeking sociological explanations for errors or failures in mathematics and its applications but not successes, or otherwise granting the primacy of mathematical reasoning in accounting for mathematical knowledge (Restivo 1993, 252). Adopting mathematics as the hard case with which to demonstrate the reach and potential of his program's claims and ambitions, Bloor latched onto Wittgenstein's illustrations of the linguistic embedding of confidence and consensus in mathematics. For Bloor, Wittgenstein demonstrated that mathematics was derived through naturalistically observable social processes of navigating and managing irreducible or undecidable conventions.

These conventions—rules and institutions, in the terms of a later Bloor book title—were not just social but *social like everything else*. In Bloor's heterodox interpretation, there was ultimately nothing intrinsically mathematical about the language-mediated determination of right and wrong in mathematics. That 'ultimately' is important, and a key source of misinterpretation of Bloor's program: this analysis focused on extremal moments of indecision in mathematics where no mathematical principle *in itself* uniquely determined the next step or inference. Asserting the continuity and primacy of social institutions in the fundamental decisions of mathematics, science, and *everything else* was Bloor's answer to postwar sociologists of science who focused on identifying the distinctive (and distinctively virtuous, rigorous, and truth-producing) features of science as a realm apart (Kaiser 1998). As Kaiser (1998) shows, this shaped Bloor's interpretations of earlier sociologies of knowledge, such as the interwar work of Karl Mannheim (e.g. 1939), to suggest misleadingly that they assumed an intrinsic difference between science and other forms of knowledge and placed science out of scope for their sociologies.

This posture of framing sociological accounts of mathematics against a backdrop of ignorance, indifference, or presumptive impossibility is itself a long-running tradition in sociologies of mathematics. Where most sociologies of modern societies could be said to have encountered situations of demystification, mathematics appeared to some to be becoming increasingly mystified. For François and Van Bendegem, "the more we have become dependent upon mathematics, mostly in an indirect or invisible way, the less we are actually understanding its principles" (2010, 122). The sociology of mathematics has been called "neglected" (Startup 1983), "severely underrepresented" (Löwe et al 2007), characterized by "a queer mixture of devotion and lack of interest" (Heintz 2000, 9; noted in Löwe et al 2007). Mathematics as a subject is, for François (2007), "untouchable and frightening" for the general public, as well as for social scientists, as a result of an alienating technical norm in global curricula. This view of mathematics as marginal to sociology (of knowledge) comes in tandem with the construction of mathematics as a limiting form of knowledge, an extreme that tests what sociologies of knowledge can say. Restivo (1985[1983], 161) called it "the ultimate arbiter of the limits of the sociology of knowledge" and noted the omission of mathematics from a popular 1970s collection on the sociology of knowledge (165). For Bloor it was "the most stubborn obstacle ... the holy of holies" (Bloor 1976, 73; see Barany 2017). For Heintz (2000, 10), the "epistemological specificities" of mathematics mean that it is not certain whether the content of mathematics can be studied through constructivist sociology.

Proposing to question the most basic and taken-for-granted elements of mathematics has put these sociologists squarely in the company of foundation-minded mathematicians themselves (see Barany forthcoming). Going back to basics, revisiting axioms, and rejustifying the seemingly obvious are time-honored narrative and mathematical techniques in both pedagogy and research. At the turn of the twentieth century, Russell and Whitehead famously and at great and inscrutable length rederived the "occasionally useful" proposition that 1 + 1 = 2. In the middle of the century, Bourbaki presented the number 1 as the product of a symbolic calculus far too tedious and complex to write out in full. The code that makes numbers work in recent computer proof systems contains multitudes, often far beneath the surface of programming interfaces that promise numerical deductions will work as expected.

This tradition is highly misleading, in similar ways, for mathematicians and sociologists alike. Most mathematics looks very little like 1 + 1 = 2, even for elementary learners. The claim that "*even this* can be understood sociologically" or "*even this* can and must be developed from more fundamental logical principles" obscures how unrepresentative *this* is for most mathematical knowledge and practice. (Barany has argued elsewhere, 2014, that in other contexts such a focus on *the very most elementary* reckoning has derived from and contributed to scientific racism.) As a result, sociologies of mathematics have tended to share many of the biases and fixations of foundational philosophies of mathematics, albeit typically from a very different angle.

Hard Cases and Soft Targets

Bloor's strong program unsettled and provoked interlocutors and critics from a whole landscape of fields from anthropology and sociology to philosophy and mathematics. Most provocative was the strong program's principle of symmetry, that the same social causes should explain true and false ideas and theories in science (see esp. Bloor 1976). If truth and falsehood were the products of social processes of investigation and consensus-making, not predetermined manifestations of natural reality, there should be no sociological difference between the success of theories now believed true or now believed false: that they were respectively believed in their own times in their own social contexts made them sociologically equivalent.

The motivation for this approach was deeply empiricist, following a scientistic commitment to the process of deriving knowledge from rigorous observation undiluted, to the extent possible, by metaphysical presuppositions. Realist truth claims were, for Bloor, metaphysical distractions from the empirical question of how people came to agree about what they agreed about the natural or mathematical world. Such questions could and should only be answered by the observation and analysis of social facts. These, Bloor joined many sociologists of knowledge in supposing, should be most visible in episodes of controversy where advocates of rival knowledges had to put their reasoning and argument in the open (see e.g. Ernest 2023).

Bloor's early articulations of the strong program included an extensive revisionist reading of Imre Lakatos's influential heuristic history of Euler's theorem about polyhedra, presented in the posthumous book *Proofs and Refutations* (1976, see Barany 2017). Lakatos's account was a prime example of the indeterminacy and revisability of less elementary mathematics in Bloor's (1976) *Knowledge and Social*

Imagery, and the centerpiece of a speculative and provocative 1978 article mapping Lakatos's historical examples of mathematical responses to exceptions, counterexamples, and other troubling phenomena to Mary Douglas's anthropology of purity and pollution. For Bloor, Lakatos's classification of responses to his titular refutations corresponded to Douglas's (1978) grid-group scheme for understanding how members of different social groups respond to disruptions to social order based on their degree of hierarchy and their concentration or mobility in or between social groups. This analysis exemplified the strong program's identification of classification as a central and constitutive activity of science, befitting Douglas's anthropological interest in the maintenance of categories as a central facet and manifestation of social order.

An appendix to Bloor's 1978 article took Lakatos's posthumous editors, John Worrall and Elie Zahar, to task for adding explanatory footnotes that toned down what Bloor saw as the more radical relativist implications of Lakatos's analysis. Worrall replied at length in the next issue of the journal, to which another shorter back-and-forth was appended (Worrall 1979). For Bloor, Lakatos's account showed the interminable revisability of mathematical concepts as well as mathematical proofs. For Worrall, Lakatos demonstrated that revisability was important in mathematics up to a point at which reasoning could be reduced to certain and rigorous logical deduction. This, to Bloor, exemplified just the sort of response to a troubling phenomenon that had been analyzed with Douglas's scheme in Bloor's article, with Worrall answering a perceived threat to the integrity of logical deduction by reclassifying disagreements about proofs as unthreatening revisions to theorems (Bloor in Worrall 1979, p. 79).

The notion—preposterous to Bloor but dreadfully serious to a number of mathematicians, logicians, and mathematical scientists—that relativist and social constructivist accounts of mathematics were a threat to logic and mathematics itself (or at least to broader support for and trust in these) became a significant component of the so-called Science Wars of the 1990s (see Harris 1999). To a degree, social constructivist challenges to the obviousness or inevitability of extremal elementary mathematics invited the criticism that they were simply (perhaps ignorantly or perhaps malignly) attempting to deny the obvious. Mathematical physicist Alan Sokal gained notoriety by publishing a hoax article in the journal *Social Text,* which he claimed had demonstrated a willingness of cultural critics to endorse any anti-scientific nonsense. Cultural critics, including historians and sociologists of science, took the occasion to historicize and sociologize the animus of Sokal and his supporters, as well as their own disciplines, much as Bloor had done in reply to Worrall (e.g. Scharping 2001).

Bloor's commitment to controversies as stages for displaying the assumptions and stakes of scientific reasoning applied reflexively to his own scholarly practice, which featured exemplary exchanges with theorists who, unlike Worrall or Sokal, shared many of his relativist and constructivist principles and motivations. A 1992 back-and-forth with ethnomethodologist Mike Lynch in an edited volume dedicated in part to staging methodological debates in the sociology of science drew out the importance of social structure and authority for Bloor's understanding of agreement and closure in mathematics (Lynch 1992, Bloor 1992). The authors revisited a parable from Wittgenstein's *Philosophical Investigations* exploring the indeterminacy of rules of counting. A pupil counts successfully by twos until 1000, then continues the series 1004, 1008, 1012 and is reprimanded for misunderstanding.

For Bloor, the pupil has shown that an infinite variety of logical rules would have been consistent with the sequence up to 1000, including an infinite number that would have the sequence continue as the pupil did. The correct interpretation was underdetermined, and is settled only by an exercise of social authority over the pupil. For Lynch, by contrast, the lesson is that rules are not reducible to their formal logical representations. The pupil has indeed erred, and this shows not that the rule was underdetermined but that there was more to the rule than what Bloor's account supposed. Following a mathematical rule is not just following (or obeying) a logical dictate; it is recognizably conforming to a pattern of observable practice. Bloor returned to the problem of rule-following at several later points, including a rich 2004 debate in *Social Studies of Science* initiated by philosopher Martin Kusch that directly revisited the 1992 arguments (Kusch 2004, Bloor 2004; Kusch opens the exchange with a review of the rule following problem in the philosophy of mathematics).

A 1999 exchange of journal articles with Bruno Latour, although not so directly concerned with mathematical knowledge, developed another dimension of Bloor's commitment to social explanation (Bloor 1999a and 1999b, Latour 1999). Latour had famously framed one of his most significant interventions in the sociology of agency and agreement in science as a generalization of Bloor's principle of symmetry. Where Bloor's symmetry principle emphasized the ultimate status of social causes to account for the stabilization of claims about nature, Latour argued that this principle of symmetry must be extended to account for the very opposition between social and natural causes. They thus differed over whether (per Bloor) symmetry was a matter of prioritizing human agency in the adjudication of claims about nature, or (per Latour) a necessary next step to account for the constructed opposition of nature and society as part of the formation of scientific knowledge.

The stakes of symmetry weigh especially heavily in strong program interpretations of mathematics. For Worrall (for example), finite logical deductions are precisely the indubitable ground truths that cannot be denied (almost by definition) by any rational person. The formal underpinning of mathematical rigor may appear to be outside the nature-society binary that concerned both Bloor and Latour in their own ways, and little in their debate would seem to contradict this impression. Latour's intervention explicitly invoked the Science Wars context of their disagreement, and the difficulty of articulating nature's role (such as it is) in a social theory of scientific knowledge. In Bloor's view, centering social explanation to the exclusion of nature was not to deny nature's relevance to scientific knowledge but to assert that, in the end, responsibility for scientific knowledge was exclusively human.

To insist on human, collective, social responsibility for the ground truths of mathematics was indeed a radical and challenging position. However, as the next section discusses, it was a position very much in keeping with a robust tradition of analysis of the social contexts and implications of mathematics, perhaps too easily missed in readings of strong program sociology outside of its formative intellectual and political contexts. Bloor built on a tradition of social study of mathematics that was based foremost in social critique rather than critique of mathematics as such, and the primacy of social responsibility was accordingly all the more visible.

Roots and Radicals

Restivo (1982, 1983) identified the formative role for sociologies of mathematics of a train of Marxist interest in the social and material underpinnings of apparently idealist mathematical knowledge. These date to Boris Hessen's famous 1931 intervention as part of the exceptional Soviet delegation to that year's International Congress of the History of Science, in London, transgressively proposing a Marxist analysis of the hosts' national hero Isaac Newton and his *Principia Mathematica*. Varying forms of Marxist philosophy and politics were important commitments for Dirk Struik (e.g. 1942, 1986) and Joseph Needham (e.g. 1956, 1959), among those Restivo identifies as forerunners of the later sociology of mathematics. Restivo also includes in this list Ludwig Wittgenstein, whose later philosophy of mathematics has been argued to have been influenced by his engagement with Marxist thinkers (see Kitching and Pleasants 2002). This was not confined to the First and Second World of the Cold War; we might add Indian Marxist mathematician and historian Damodar D. Kosambi as a contemporary deeply invested in materialist historiography.

At one level, the significance of Marxism to the social study of mathematics derives from the philosophical commitment to understand the development of ideas and production of knowledge in social, economic, and material terms, as part of the global capitalist system. But, especially for the postwar contributors to the Marxist tradition of social studies of mathematics, the dimensions of societal direction of and social responsibility for mathematics motivated inquiry into how mathematical knowledge responded to social contexts and conditions. Such "social and economic connections" and their relevance to questions of responsibility and the role of mathematics in modern society were central motivations for later work that somewhat critically recognized the influence of Marxist perspectives (e.g. Bos and Mehrtens 1977, pp. 10-12).

Restivo (1983, see also 1992, 1993, 2001) embraced the strong program as a foundation for continued materialist investigation of mathematics. Having engaged in detail with existing Marxist sociology of mathematics which he argued had been insufficiently grounded in materialism, Restivo took sociological inspiration from Spengler (1926) whom he identified as the first sociologist of knowledge to have expressed a strongly constructivist view of mathematics as a human activity inherently defined by its social, cultural and historical milieu (1983: 86). Restivo recognized the "sociological imperative" to understand mathematics as a social pursuit (1993: 249) and responded by laying down an ambitious program for a radical sociology of mathematics "in which all talk about mathematics is social talk." This built upon the Marxist premise that all science is a collective and socially embedded activity and drawing on essential social elements of scientific activity, such as language, other "material", and the scientists' existence within multiple layers of social institutions. Restivo shifted the focus from the individual mathematician to the "math world": a collective representation of the activities and institutions inhabited by mathematicians. Restivo's key argument was that mathematics is not manufactured by individual mathematicians alone but is a product of "math worlds". These "math worlds" are embodied by mathematical objects and are inseparable from the social history of their own construction. Mathematical objects, in turn, are collective representations, sensibilities and worldviews—and not merely symbols.

This, for Restivo, showed the necessity of "entering math ethnographically" (1993: 249). Restivo's later work defends the sociology of mathematics from "misguided" fears of its relativism when in his view it is in fact realist (2001). Restivo brushes aside the "unreasonable effectiveness of mathematics" conundrum as something that in fact demonstrates the realism of his sociology of mathematics: he declares the success of mathematical applications in the "real world" unsurprising, since mathematics is "'borrowed' from the real world to begin with" (1985, 180-181, see also Hamming 1980). Restivo elevates mathematics, along with logics and religion, to the status of a "battleground" (2008) similar in scope to the Copernican revolution, and highlights the challenges math-fueled technological progress can bring if not accompanied by sociological understanding of mathematics as a social act and a complex of social institutions (see also 2016, 2017).

The critical and transformative potential of social accounts of mathematics has been especially visible in mathematics education and in studies motivated by that field. Restivo and Bloor both offer social constructivist foundations for philosopher of mathematics education Paul Ernest's examination of learning, language, and ethics in mathematics (e.g. Ernest 1991, 1994, 1998, 2000, 2018; see also Ernest's chapter in this handbook). Leone Burton (e.g. 1995) outlines a feminist epistemology of mathematics and mathematics education. The visible and large-scale social contexts and consequences of mathematics education, especially primary mathematics education, have motivated social studies of race and marginalization in and beyond mathematics classrooms (e.g. Gutiérrez 2013, 2018, Martin 2019). Sociological critiques have addressed institutional inequalities and injustices based on race, class and gender, in various settings: elitism in academic journals (e.g. Chang and Fu 2021), racist political regimes (e.g. Mehrtens 1987), Hunsicker and Rittberg's discussion of the social injustices caused by unequal dissemination of mathematical ideas (2022), or the unequal effects of mathematization in different applied contexts (e.g. Ziols and Kirchgasler 2021), to name a few examples.

Critiques of school mathematics and the particular eurocentric model on which many hegemonic curricula have been based have underwritten programs of investigation into other forms of mathematical learning and activity under the rubric of ethnomathematics (e.g. d'Ambrosio 1997, 2000). Emphasizing the diversity, contextual specificity, and mathematical substance of knowledge and practice across different places and by historically underrepresented and underprivileged people, ethnomathematical research has ranged across non-Western cultures as well as examples of activity in Western settings that has not traditionally been understood to be mathematical but can be interpreted from a mathematical perspective (e.g. Lave 1988).

Gellert and Jablonka (2007) approach mathematics sociologically by locating mathematical knowledge in power relations and social inequality. They link and problematize the concurrent trends of mathematization and demathematization in contemporary society and warn of the resulting political "threat to democracy because of a widening gap of mathematical knowledge between constructors and consumers". Williams and Wake (2007), employing a microsociological and ethnographic approach to inform their analysis, similarly find that mathematical knowledge in workplace division of labor is increasingly "black-boxed, with groups of people excluded or insulated from mathematical knowledge and power" (2007). François (2007) argues that ethnomathematics has the potential to play a transformative role in Western school curricula, help undo some of the social stratification perpetuated by existing mathematics education and demystify the "untouchable and frightening" public perception of mathematics. Drawing on Alan J. Bishop's "M-m distinction," she argues that school curricula reproduce social stratification by restricting vocational pupils' exposure to "small m mathematics" ("a purely deductive science, using a technique-oriented curriculum" and basic competencies such as counting, designing, explaining, locating, measuring) while far fewer pupils have easy access to Mathematics (the Western scientific discipline, including ideas of mathematical beauty and creativity).

Watching and Listening to Mathematicians

Sociologists of mathematics have long struggled to catch mathematics in the act. Even when one can directly observe people at times and in places where they are doing mathematics, seeing the mathematics being done as such can be vexingly difficult. Mathematical communication combines visible, audible, and other sensory qualities with elements of invisible cognition that can be necessary to make sense of the more observable aspects of such communication. Proxies, inferences, and reconstructions can make up for some of this inaccessibility, while bringing their own compromises and challenges.

At one extreme end of the scale of observation, cognitive scientists Lakoff and Núñez (2000) have contributed to materialist understandings of mathematics by correlating specific mathematical concepts and intuitions to features of embodied cognition. Their universalist conclusions about undifferentiated masses of human beings, and their corresponding generalizations about mathematical knowledge, are by design and supposition un-sociological, while sharing sociologists' interest in the tangible human production of mathematics and resisting idealist notions of mathematics as entirely separate from the human condition.

Drawing on ethnomethodological approaches to scientific knowledge (e.g. Livingston 1986), Greiffenhagen (sometimes in collaboration with Sharrock) has expanded the scale of observation from that of individual cognition to small settings of interpersonal communication including university classroom lessons and supervisor-student meetings (Greiffenhagen 2014; Greiffenhagen and Sharrock 2011, 2019). Barany and MacKenzie (2014) combined ethnographic observations with interviews to characterize the material production of mathematical knowledge at the scale of small research communities, identifying the situated and dynamic roles of particular forms of writing (on blackboards, scrap paper, and other media) for mobilizing particular kinds of understanding and communication. Kremakova (2016a, 2016b) used ethnography and autoethnography to uncover links between mathematicians' careers, professional identities, and mathematical activities such as teaching, learning, collaboration, proving, publishing, supervising graduate students, and so on, locating these in the context of ongoing marketisation of higher education and the precarious scientific labor market.

Methods of institutional sociology permit indirect characterizations of mathematics and mathematicians at larger scales. Early examples can be found in Hagstrom (1965), Fischer (1973) and Startup (1979), who investigated the composition of mathematics and other scientific university departments and the social characteristics and working lives of academic mathematicians, reflecting on (and analyzing mathematicians' own reflections of) aspects of mathematical labor and practice along the way. More recently, combining surveys, publication data, and other methods, a large international collaboration to study gender gaps in science has produced large-scale systemic findings about the contemporary mathematics profession (e.g. Mihaljević and Santamaría 2020, Steinfeldt and Mihaljević 2023).

Sociologists of mathematics have also used variations on historical methods, focusing on discursive and contextual analyses of written records to watch mathematicians indirectly by viewing the records of their written communication with each other. Such studies have ranged in scale from Merz and Knorr Cetina's (1997) examination of the email records of a single collaboration, to Rosental's (2008) analysis of usenet discussion posts about a contested theorem in mathematical logic, to Martin and Pease's (2013) study of online mathematics interactions including question and answer forums, to Steingart's (2012) examination of the paper trail of a large-scale and long-running project in group theory. Written records can, through their references and omissions, show aspects of unwritten practice and indicate how mathematicians rely on combinations of formalized and unformalized reasoning to make sense of concepts and arguments, and to establish confidence or articulate its absence whether or not these make sense.

A vital source of knowledge about mathematical knowledge has been mathematicians themselves. In the course of their mathematical practice and careers, mathematicians necessarily develop personal and shared understandings—to varying degrees of explicitness and formality—of relationships between mathematical institutions and mathematical knowledge. Many core activities of mathematical work, from evaluating research proposals and results to assessing peers and students to navigating careers and organizations, all depend on sociological knowledge. Restivo (1993, 248) calls this "[t]he everyday folk sociology of mathematicians." Beyond their personal navigation of their professional worlds, some mathematicians pursue (with varying degrees of rigor and amateurism) social and humanistic studies of their subjects. These can all be significant sources of insight and understanding in the context of social theories and more systematic sociological analyses of mathematics (e.g. Lane et al 2019), while being susceptible to the biases and limitations of their informants (see Hanna and Larvor 2020). Socially and politically engaged mathematicians have employed sociological theories in support of both theoretical and political arguments against institutional racism and other sources of inequality and discrimination in academic mathematics departments (see e.g. H[Harron] 2017).

Many academic mathematicians engage in sociologizing their "math worlds" and turn to insights from anthropology, sociology, history and philosophy of mathematical practice to make sense of the mathematical experience. Mathematicians have produced social commentaries on various aspects of the mathematical existence, work, career, biography, professional identity, teaching, the nature of mathematics and the way it is done (see e.g. Hardy 1940, Wilder 1950; Hersh 1981, 1997, Thurston 1994, Lockhart 2002, Harris 2015). Mathematicians' endogenous sociologies proliferate in global public channels such as blogs and public talks, steering public discussions in the mathematics community and beyond (Terence Tao, Timothy Gowers, Edward Frenkel, Marcus du Sautoy, Günter M. Ziegler, Piper H[Harron], to name a few). These voices, often in dialogue or open disagreement with one another, serve the dual purpose of being an ongoing participatory sociology of mathematics produced by "mathematical natives" in its own right and forming a metacorpus of data for nonmathematician sociologists of mathematics to interrogate and draw upon. These "native sociologies" also provide a key point of tangency for sociologists and mathematicians at which mutual assumptions, biases and ignorances become exposed, helping to develop a more realistic mutual understanding of what it is that mathematicians, and sociologists, actually *do*.

Extensive conversations with mathematicians formed the basis for Bettina Heintz's (2000) study of contingency and flexibility in the "inner world of mathematics," which sets mathematicians' perceptions of their principles and practices in the context of sociological theories including those of Bloor and Restivo. Lane et al (2019) apply Ingold's notion of crafting as wayfaring to analyze mathematical metaphors of landscape, journey and craft to understand the way in which mathematicians' work is represented and contrast the realities of mathematicians' work with the myth of the lonely genius. In Kremakova's (unpublished) sociology of labor and career patterns of mathematicians, personal experiences and perceptions of career trajectories among mathematicians aggregate into a systemic view of patterns of research and mobility with significant social and intellectual consequences. Kremakova's approach of analyzing what mathematicians do as scientific labor reveals tight links between what happens inside mathematics as a scientific discipline (such as the content of emerging mathematics, the formation of university curricula, or the choice of research questions and methods related to available funding), on the one hand, and the workings of scientific institutions and the conditions of the scientific labor market, on the other. Analyzing the mathematician as his or her own "nomological machine," Kremakova discusses mathematicians as embodied, material and social beings, working with material (or quasi-material) artifacts, and constantly negotiating tensions and conflicts between individual career paths and constraints within mathematical institutions.

Important constraints which define and anchor the mathematical experience in socio-economic realities include the precarity of early career work and contracts, the need to reconcile personal life and caring responsibilities with the challenges of standardized academic career trajectories tailored to an idealized atomised and disembodied rational agent, and other challenges posed by the "new spirit of academic capitalism" (Kremakova 2016b). Narratives of inspiration and passion prevalent in mathematics, as well as the challenges of quantifying and demonstrating mathematical "outputs" make mathematicians (similarly to other academics) prone to exploitation and self-exploitation as workers embedded in contemporary capitalism. The "technocratic time" of contemporary neoliberal academia allows little room for "playful exploration" and insufficient "thinking time" required to do *new* science, while the current "managerial turn" in universities (characterized, inter alia, with increased reliance on productivity metrics for research assessment) produces strategic responses by mathematicians who, in order to remain successful in their field, are led to turn more into managers and less into artists or adventurous explorers. As a result shorter-term research programs become prioritized, pragmatic publication strategies are adopted, and "safe choices" (of which open problems to pursue, or of which candidates to hire in mathematics jobs, for example) are made. The precarious conditions and increasing internationalization of the job market for early career mathematicians are hard to combine with a viable (gendered) work-life balance and also contribute to the prioritization of short, "doable" research projects—or even projects which have already been carried out but are as yet unpublished—as mathematicians without stable positions find themselves unable to commit to and carry out ambitious long-term research agendas. The (perceived and real) lack of stability, the pressure to publish fast, and move countries for work create insurmountable barriers for mathematicians with caring responsibilities (disproportionately female), disabilities, lower socio-economic backgrounds, and results in perpetuating multiple social inequalities within the mathematics community.

At the same time, mathematics "in the making" also brings moments of joy and other intense emotions. Mathematicians experience and talk about playfulness, creativity, pure curiosity, intellectual satisfaction, the addictive reward of sudden "a-ha moments" following long periods of hard toil, the skill of making mistakes without embarrassment, the pride in developing your own mastery and seeing your students surpass you, the joy of collaboration and belonging to a shared community of practice and of values (both in person and across space and time). Mathematics is made up of a wide variety of social roles valued by the community. Mathematicians devise, maintain and reproduce collective institutions, roles and myths in service of their own community: examples include the "troll on the bridge" (the head of department who craftily uses university leadership to shield department members from excessive bureaucratic demands), funding councils and journals which legitimize and uphold the importance of various strands of mathematics against the tyranny of fashionable buzzwords, or outreach programs targeting marginalized groups both within research communities (conferences such as "Women in Numbers" and organizations such as the Network of Minorities in Mathematical Sciences) and among the general public.

Conclusion: Making and Breaking Order

Sociologies of mathematics aim to comprehend the collective achievement of order and agreement in mathematical knowledge and mathematical institutions. The orderly production of theorems and the orderly production of mathematicians to produce, store and share them are, in their own ways, distinctive social problems addressed by common social forms in the modern world. Sociologies of mathematics matter because these mathematical orders are never as stable as they seem, and that instability can present risks and opportunities that run through and beyond mathematics as a discipline, subject, and form of knowledge.

We note and anticipate productive scholarship in these directions at the intersections of traditional and new subfields, such as the sociology and anthropology of work and labor, of gender, of social inequality and of education, higher education and academia, feminist sociologies of care and social (re)production, and the sociology of technology and computing.

Institutional understandings of mathematics are especially important in the midst of a long era where the uses of mathematics and the processes of producing that mathematics have each grown more complex and more consequential across greater segments of the social world. Algorithmic and informatic mathematics draws a great deal of attention in this regard, but even seemingly esoteric, isolated, or pure areas of mathematics participate in a political economy and social arrangement of making, knowing, and sharing mathematics that conditions its higher-profile manifestations. The institutions of mathematics span modern societies and involve many interconnected and hierarchical relationships of knowledge and power with profound implications for what mathematics may do in the world, how, by whom, for whom, and to whom.

It therefore bears emphasizing, to an extent not emphasized in the foregoing analysis, that mathematics in its institutions *and* as a form of knowledge is today (and long has been) in many respects a tremendous locus of inequality and injustice, a resource for the empowered and a burden for the marginalized. What we have, rather, emphasized in the foregoing is the extent to which concern for the social ramifications of mathematics has motivated and guided investigations into its social and material foundations, and the potential for such investigations to revise what is taken for granted about what may and must be in the world of mathematics and from mathematics in the world.

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