Of Polyhedra and Pyjamas: Platonism and induction in meaning-finitist mathematics

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Abstract
This chapter considers the epistemology of mathematical concepts through classical Edinburgh School *meaning finitism*, a philosophical principle from the sociology of knowledge that is now several decades old. Meaning finitism is based on the idea that communities of people adaptively derive open-ended meanings and classifications from a finite basis of experience, choosing how to interpret the world by revising their past interpretations through social interactions. Using Barry Barnes's metaphor for classification as hospitals issuing pyjamas alongside Imre Lakatos's famous study of Euler's theorem about polyhedra, I analyze the implications of this school of meaning finitism for the problems of epistemic induction (inferring about the future based on past experience) and Platonism (assuming the existence of ideal objects independent of concrete experiences) in the elaboration of complex mathematical concepts from simple models and examples.

Keywords: Sociology of Scientific Knowledge (SSK), Edinburgh School, meaning finitism, epistemic induction, mathematical Platonism, labeling, models, examples, heuristics.

Introduction

> Think not of a word being uttered, vibrating through the air, and being lost, but of the farmer clipping the sheep's ear, the nurseryman tagging the plant, the hospital issuing pyjamas.
>
> (Barnes 1983, 528)

Near the start of his seminal essay on "bootstrapped induction," Barry Barnes asks the reader to think of labeling as an active enterprise, one which makes "an enduring change to the situation" (ibid.). Labels, he explains, are not passive, fleeting descriptions. Rather, they stay with their subjects like pyjamas on hospital patients, situating them in patterns of language and cognition. These "pattern attachment systems" are what enable the objects of the world to become concepts that are circulated in a social field of knowledge. Whatever objective order they may independently possess, objects cannot be ordered in the conceptual universes of the people who study them without such systems of labels.

Though mathematical concepts can be inspired by the things of the natural world and can be the basis for interventions in the world, mathematical concepts cannot bleat in a field or be arrayed in greenhouse rows. The things of mathematics appear purely through human activity, manifested in definitions, textbooks, diagrams, gestures and utterances by and for people. Though few would suppose that mathematical concepts have the kind of existence of a tree in a forest, which one supposes could grow, photosynthesize, and fall (perhaps even making a sound) without any human intervention, there are nonetheless features of mathematical experience that suggest mathematical concepts, once formulated, do take on a certain life of their own or might even have been "out there" waiting to be formulated.[1]

This peculiar ontological character attributed to mathematical concepts raises a host of epistemological questions that are related, but not typically reducible, to corresponding questions about scientific and social knowledge. One important perspective on such questions has been the strong program of the Edinburgh School of the sociology of scientific knowledge (SSK), associated with two of its chief proponents, Barry Barnes and David Bloor. SSK debuted in the 1970s and 1980s as an exciting and sometimes controversial approach to the study of science in relativist terms, with roots in the philosophy Ludwig Wittgenstein. It featured significantly in the so-called "science wars" of the 1990s, and has since waned in prominence, though it continues to figure in academic curricula, research, and debates in the sociology of science.

Emphasizing the social life of labels, advocates of the strong program in SSK stress the irreducibly contingent human aspects of scientific and mathematical knowledge. By casting knowledge as fundamentally relational and revisable, the strong program's adherents instructively contrast their accounts against those that place greater weight on the supposed power of mathematical entities and inferences to compel assent in and of themselves. That is, where many philosophers aim to account for the appearance and implications of determinacy and certainty in mathematics, SSK guides one instead to emphasize the sources of *indeterminacy* in mathematical understanding and how those indeterminacies are socially resolved.

In this chapter, I place classical accounts of mathematical concepts in the context of the Edinburgh School's social epistemology, exploring (in Barnes's metaphor) what it means for a polyhedron to be issued pyjamas. I begin by sketching the principles of meaning finitism in the strong program, and then illustrate the meaning-finitist account of mathematical objects by considering Imre Lakatos's (1976) depiction of Euler's theorem concerning polyhedra. Using this framework, I address in turn the relationships between meaning finitism, epistemic induction, and mathematical Platonism, which imply a special importance for simple, workable models and examples. While meaning finitism is foremost a principle about labeling and classification, I explain how a meaning-finitist account of these activities applies more broadly to the use and revision of mathematical concepts.

I conclude by indicating how Barnes's pyjama metaphor can be extended profitably to account for the process, less considered in SSK accounts of mathematics, by which simple mathematical concepts are used to establish more complex ones. This view expands the SSK account of meaning finitism from a theory about the conditions of stabilization for simple mathematical concepts into a theory that also encompasses their conditions of change. Barnes's pyjama metaphor, I argue, impels us to examine the figurative seams of mathematical practice, how concepts are negotiated and stabilized. It is not enough to describe exemplary mathematical concepts as wearing pyjamas without also asking how those pyjamas are issued, made to fit, and occasionally stripped in the process of mathematical knowledge-making.

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[1] This idea forms a central theme of Hacking (2014), a distinctive recent entry in an enormous body of scholarship on this question. Hacking argues, in part, that these features of mathematical experience are not as representative of what mathematicians spend their time doing as philosophers frequently suppose. However, the many applications of mathematical reasoning and representation in other areas of mathematics, science, and beyond, still seem to demand a philosophical understanding of mathematical ideas as less-than-arbitrary.

Strong Program Meaning Finitism

Meaning finitism, within the strong program of SSK, is based on the idea that "concept application is a matter of judgment at the individual level, of agreement at the level of the community" (Barnes 1982, 30). This entails five central claims, summarized by Barnes, Bloor, and Henry (BBH 1996, 55-59):

1. The future applications of terms are open-ended;
2. No act of classification is ever indefeasibly correct;
3. All acts of classification are revisable;
4. Successive applications of a kind term are not independent; and
5. The applications of different kind terms are not independent of each other.

The first three claims describe knowledge as *flexible* and the final two emphasize that it is also *relational*. The flexibility of knowledge comes from its fundamental formal indeterminacy: every thing has similarities and differences with every other thing or collection of things, so in no case do our past classifications logically and indefeasibly compel us in our future ones (BBH 1996, 51, 78; Barnes 1982, 28-30). Some classifications are more defensible than others, but it is always possible to imagine that some (perhaps bizarre) method of identifying and weighing similarities and differences could overturn even the most obvious-seeming divisions between natural objects. Meaning finitists reject the notion that there are pre-given natural partitions of the world according to different kinds (Barnes 1981, 315).

Instead, classification must proceed by "analogy between the finite number of our existing examples of things and the indefinite number of things we shall encounter in the future" (BBH 1996, 51; see also Barnes 1982, 49; Bloor 1983, 95). Old classifications form the basis of new ones according to their degree of similarity to the objects in question, something which is always negotiable but never purely arbitrary (Barnes 1981, 309, 312; Barnes 1982, 29; Bloor 1997, 10, 70). The context-dependent negotiations over classification are the basis for the social study of meaning (Barnes 1981, 314; Barnes 1982, 30; BBH 1996, 79).

In the terms of Barnes's metaphor, a particular set of pyjamas can fit people in a variety of shapes and sizes, and an individual person can fit (to varying degrees of comfort) in a few different sizes and styles of pyjamas. If you are currently wearing hospital pyjamas, you are more likely to be issued them again in the future, but you are not consigned to a life of loose-fitting hospital garb. When a new patient arrives at the hospital, that patient's similarity to other patients can guide which pyjamas are issued, but there may be multiple sets that fit. Pyjamas, meanwhile, collapse distinctions between those wearing them, transforming distinct individuals into common patients (or patients of a certain size or shape). They make patients more uniform by temporarily covering some of their features, and by equipping them with a shared and recognizable sartorial marker.

Mathematics has a special place in the strong program for both epistemic and historical reasons. Epistemically, it differs from the natural sciences in treating what is supposedly a realm of pure ideas. This requires a reinterpretation of ostention, the ability to name a thing by gesturing at it, which has a foundational status in meaning-finitist epistemology (Barnes 1981, 306, 308; Barnes 1982, 35). In classical meaning-finitism, the thing you are classifying is itself something at which you can point. But in mathematics, anything at which one can physically point is already at least a step removed from the mathematical ideals whose classification is at issue. To what exactly, in mathematics, would pyjamas be issued?

Historically, mathematics has represented an ideal of knowledge independent of messy human contingencies. Paradoxically, it has been imagined to be free of *both* nature *and* culture. This presents a double challenge to the fundamental approaches of SSK, which stress the interlocking roles of nature and culture in human knowledge. For this reason, Bloor (1976, 73) called mathematics "the most stubborn of all obstacles to the sociology of knowledge … the holy of holies."

Thus, taking a note from the later aphorisms of Ludwig Wittgenstein, strong program scholars began by attacking the self-evidence and inevitability of some of the most natural-seeming mathematical practices—including counting by twos and taking twice two to equal four (e.g. Bloor 1973, 1983, 1997; BBH 1996).[2] Somewhat less discussed but nonetheless canonical, Lakatos's (1976) heuristic history of Euler's theorem offered the strong program's expositors a chance to extend the case for meaning finitism to less elementary mathematics (e.g. Bloor 1976, 130-137; Bloor 1978, 248-250; BBH 1996, 187). According to Euler's theorem, the number of vertices plus faces minus edges of any polyhedron is equal to two. Lakatos showed how, starting at the turn of the nineteenth century, attempts to prove this theorem involved confrontations over the very meaning of the term "polyhedron," among several contested mathematical concepts.

In Bloor's gloss on Lakatos's story, Euler's theorem began as an observation about the relationship among vertices, edges, and faces in a certain limited collection of polyhedra. This observation was generalized by Euler into the equation V-E+F=2, an example of epistemic induction (Bloor 1976, 135). That is, based on a limited collection of observations, Euler stipulated that one could expect his formula to hold whenever one counted the vertices, edges, and faces of a polyhedron in the future. Lakatos's story unfolds in a series of historically proposed counter-examples—situations in which that expectation that the formula would hold did not seem to bear out—each of which complicates a different aspect of proofs of the theorem or definitions of polyhedra (ibid.: 133-134). Because, as Bloor asserts, "Polyhedra have no essence," these counter-examples are accepted or rejected according to social interests, and the decisions "will reveal what types of figure and what features of figures are held to be important and interesting" for different mathematicians (ibid.: 135-136; see also Bloor 1978; Barnes 1981, 325).

Where mathematical concepts and proofs are often treated as though they are stable and unambiguous, Bloor takes Lakatos to show that "informal thought can always outwit formal thought" (Bloor 1976, 137). As Wittgenstein held for his more elementary examples of mathematical practice, the apparent self-evidence of an example or figure is instead the result of training and conventions (BBH 1996, 182-183). That is, self-evidence must be replaced by social training if one is to understand how mathematicians agree about an object or inference. People do not arrive at the hospital already wearing perfectly-fitting pyjamas, and the objects of Lakatos's narrative do not enter the discussion obviously or inevitably as polyhedra or counterexamples to Euler's theorem. It is the hospital staff's or mathematician's training and experience that helps them say (contingently, as each case arises) what is what. The processes of establishing mathematical definitions and procedures are thus viewed as coordination problems

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[2] These studies formed the core of the strong program account of rule following. See Bloor 1992 and 1997; Lynch 1992a and 1992b. It is also possible to challenge these seemingly natural practices historically, as I have done in Barany 2014, which shows the arbitrary (and, indeed, deeply racist) development of ideas about the naturalness of certain approaches to counting and basic arithmetic in the second half of the nineteenth century.

subject to social explanations. The mathematicians in Lakatos's tale always work with polyhedron-concepts derived from a finite stock of examples, principles, and intuitions, and so both "the counter-examples and the proof-idea had to be actively brought into contact with the concept of the polyhedron" (Bloor 1976, 139).

By focusing on the dialogical elaboration of the concept of polyhedra and a theorem about it, Lakatos's story exemplifies what meaning-finitist SSK takes to be the central process of mathematical knowledge-making. Here, polyhedra are understood in terms of a set of instances, what Barnes calls the concept's "tension" (Barnes 1981, 308). The tension includes at different times many kinds of images of certain exemplary polyhedra, including those classified as cubes, prisms, and tetrahedra, as well as formal rules or definitions like "a solid whose surface consists of polygonal faces" (Lakatos 1976, 14) and heuristics or principles such as "objects to which Cauchy's proof applies." There are necessarily only finitely many elements of this tension, and Lakatos's narrative can be read as a battle over what should properly be included. Every mathematical claim, here, is an assertion about what it means to be a polyhedron, and thus how the classification of "polyhedron" should apply. Conversely, when we talk about polyhedra (in general) we are implicitly talking about whatever composes the concept's tension, just as we speak of all the individuals wearing hospital pyjamas when we discuss patients in the abstract.

A new proposed definition or counter-example may be disqualified from deserving the title of polyhedron, though each one Lakatos introduces can plausibly be either accepted or rejected (meaning-finitist claims 1 and 2). Figures that were once deemed polyhedra and proofs and definitions that were once thought to apply to polyhedra can lose that status in light of new examples or arguments (claim 3). In each case, new examples are evaluated in light of existing definitions and intuitions for what a polyhedron is (claim 4), and the examples are often also contrasted with or judged with respect to other related mathematical terms, like edges or surfaces (claim 5). When a rule or image is classified with the term "polyhedron", it is issued a pair of (always revocable) polyhedron pyjamas which enroll it in subsequent disputes over the status of polyhedra. Such a classification changes how these objects are viewed, understood, and used, temporarily emphasizing or attributing some of each object's features while obscuring others.

Induction

Thus, in place of ideal objects to be described by rules and illustrated by examples, the meaning-finitist account of mathematical objects gives us a heterogeneous constellation of images and formalisms. Conventional accounts of mathematics tend to take a top-down approach to the epistemology of mathematical objects, putting ideals at the center and asking about the relationship between such ideals and what we know about them. By contrast, meaning-finitism insists upon a bottom-up approach, asking first how we know about objects and only later (if ever) inquiring after what those objects really are, if indeed they can be said to have an existence beyond what we know about them. Where a conventional account of Lakatos's story would say that different understandings and definitions of the same ideal polyhedra changed or were disputed, meaning finitists would find it unnecessary to stipulate the ideal polyhedra at all, and would instead make those changing understandings the center of their version of the story. For the latter, the pyjamas make the polyhedron.[3]

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[3] I thank the editors for underscoring the analogy latent in the "pyjama" interpretation of polyhedra to the maxim that "the clothes make the man."

Here, objects lose all connection to the ideal world and are instead examined solely in terms of how they are understood in the context of mathematical arguments. This is why Bloor asserts that "Polyhedra have no essence." Whether or not they have an essence in some ideal realm, his interest is in what we can know with certainty about polyhedra in the human world. Within meaning finitism, this means that the philosophical study of mathematical objects, which might include both ontological and epistemological concerns, becomes solely an epistemological inquiry into mathematical knowing to the exclusion of ontological questions about mathematical being. Even if we care foremost about what is underneath the pyjamas, the pyjamas are what we see, and hence what we study.

If all we care about are the acts of labeling associated with a concept, the concept itself seems to slip from view. Meaning finitists see it differently: if all we care about are the acts of labeling associated with a concept, then we may as well speak of concepts and their associated labels interchangeably. Because labels are always changing to incorporate new understandings and examples, this means that concepts, too, are always changing. The implications can be counter-intuitive. Even though we may interact with people differently when they are issued (or not issued) different pyjamas, we do not usually think of those people as themselves changed by their clothes. Meaning finitists ask us to see a person in one set of pyjamas as different from that "same" person in a different outfit because we interact with that person differently. People start to seem less like stable individuals and more like shape-shifting products of superficial interactions. The corresponding implications for the (in)stability of mathematical objects help explain the hostility toward the strong program from many philosophers of mathematics, whose discipline was in many respects founded on the distinction between objects and their representations.

On the one hand, it is not necessarily strange to think of mathematical concepts as existing in a constant state of flux. In Lakatos's tale, processes such as "monster barring" allow mathematicians to exclude examples from consideration, and "monster adjustment" allows them to domesticate examples to fit certain formal specifications by re-casting their salient features (Lakatos 1976, 14-23, 30-33). As Lakatos makes clear, something that counts as a polyhedron for some people at one point in time has no guarantee of counting as one for different people or at different times. Disagreements about classifications and properties of specific postulated examples show that even if there were a "right answer" that existed outside of mathematical negotiations, it would not be decisive for the historical elaboration of a theorem like the one Lakatos describes.

But while what counts as a polyhedron is not fixed for Lakatos, the formalisms of mathematics appear relatively more so. The process of proofs and refutations, for Lakatos, is a dialectic whose end is formalisms which are better able to circumscribe the shadowy apparitions of the informal polyhedron-concept. Lakatos thereby introduces an asymmetry, between informal and formal objects or concepts, that appears at odds with the tenets of the strong program of SSK, which assert that *all* objects comport to the same social rules of classification. A more detailed elaboration of how mathematical objects are developed through mathematical practice is needed to account for this apparent asymmetry under the uniform rubric of meaning finitism.

All knowledge, for meaning finitists, has the same basic starting point: "Induction is constitutive of human thought at every level" (Barnes 1974, 9; see also Bloor 1976, 118; Barnes 1981, 320). Because "there are no terms for which meaning or use is self-evident," even mathematical terms must be developed from something that is, of necessity, non-terminological (Barnes 1982, 26-27). No matter how complex, every concept can be traced to some primary acts of ostention, where initial terms are baptized in relation to elements of one's immediate experience. That is, words, objects, and experiences are neither self-generalizing nor self-defining, and must be understood in terms of other words, objects, and experiences. One's experience with the body of phenomena we call the physical world undergirds a "highly elaborated world-picture" containing meanings, practices, and conventions "only tenuously connected with what can fall within anybody's experience" (Bloor 1976, 86; Bloor 1983, 91; Bloor 1997, 39). Our concept of circles, for instance, comes about only through repeated encounters with definitions, illustrations, properties, and proofs (see BBH 1996, 63-64). As Lakatos illustrates for polyhedra, this process of elaboration can in principle be open to contestation at every step, and, indeed, it is often that very contestation that drives the elaboration forward.

Patterns in our experiences allow us to form generalizations. In this view, Euler's theorem that V-E+F=2 generalizes the expectation that whenever we encounter a polyhedron it can be found to satisfy the relation V-E+F=2. (Note the active "can be found to" in place of a passive "will"—having a mathematical property is always the result of an active determination.) This is a clear example of epistemic induction, anticipating that the future will conform to the patterns of the past. In mathematics, as in most things, such an induction proposition is generally tacit. Induction concerns assessments of probability and confidence in generalizations (Barnes 1981, 318-319). Mathematics, the art of making particularly confident assertions about particular kinds of patterns, is thus an extreme example of the inductive thinking at the heart of all concept-formation.

Of course, new information and new understandings can alter the meaning of a theorem like the one Lakatos considers. Despite its changing meaning, we can still understand it as the same theorem from one moment to the next. This continuity is maintained in two basic ways. First, one can often match new objects to the pattern "polyhedron" with little difficulty, and can confirm (or challenge) the inductive theorem on the basis of that identification. Though the theorem may have been modeled with a picture of a tetrahedron, a picture of a cube or triangular prism will not trouble an ordinary attempt at verification. But there remain cases where this pattern-matching is less clear. In this second situation, the theorem is modified or upheld by establishing a convention regarding the new object (or alternatively by modifying the theorem), as when Lakatos's characters debate whether a new proposed counterexample is really a polyhedron.

Concepts, like the theorems about them, follow the same bipartite system of maintenance. Thus, Lakatos's characters most often count vertices, faces, and edges routinely—there is little dispute about simple tetrahedra or other familiar shapes. Quite frequently, however, objects are much harder to recognize unequivocally. Confronted, for instance, with polyhedra that have stars rather than convex polygons as faces, Lakatos's characters undergo a process of monster adjustment, arguing over different ways of identifying what an edge or a face really is with respect to a specific example. These two systems of theorem or concept-maintenance represent two kinds of classification activity in strong program meaning finitism, drawing primarily on either observable pattern matching based on identifiable features of objects (natural-type) or social negotiation based on conventions of identification and signification (social-type).[4] Some

[4] Barnes calls these, respectively, P/N devices and S/S devices (Barnes 1983, 530). Bloor (1997, 40) goes further to distinguish pattern-matching activity from its mathematical interpretation, in which case all that is properly mathematical is maintained through S/S devices. As with the view

people—perhaps those wrapped in bandages—just look like hospital patients, even before you issue them pyjamas. Some people require examination and fitting, and may require that the pyjamas be altered in some way, before the hospital clothes will be both a social and sartorial fit.

The asymmetry in Lakatos's treatment of formalisms derives in part from this distinction. Where assessments of mathematical statements appear to take the form of a natural-type classification, the flexibility and conventionality of these statements' interpretations is elided, in part, by the success of the pattern-recognition. Through much of Lakatos's book, his characters work to establish ultimately arbitrary conventions in order to clarify concepts that are initially less formal. These appear to readers as social-type classifications because it is easy to identify the presence of ambiguities and competing interpretations. On the other hand, activities like counting and identifying edges and vertices are particularly well practiced and rarely controversial (even if used to innovative or unusual ends), so formal propositions involving these terms have a strongly natural-type appearance, even where they may have been initially bootstrapped and maintained by social-type activities.

Formalisms thus achieve their apparent stability through the naturalization of social-type classification into a counterfeit of natural-type classification. For instance, that the results of an algebraic deduction appear inevitable to trained mathematicians is seen to be a natural property of algebraic formalisms rather than the result of a learned system of conventions and manipulations that could have been otherwise. The case of star-polyhedra and the technique of monster adjustment from Lakatos's narrative, however, show how even firmly stabilized patterns can be tested and reconfigured (e.g. Lakatos 1976, 16-17, 30-33).

Here, examples and models have an especially important place. They are the basis of both learning old concepts and developing new ones (Barnes 1982, 18, 52; BBH 1996, 102-103, 105; Bloor 1997, 11). Because pattern matching and analogy in abstract mathematics are based to only a limited extent on perceptual judgments, there are more opportunities for intervention from social-type stipulations of similarity or dissimilarity (BBH 1996, 106-107). Whether an object gets to wear pyjamas in mathematics depends more on what we decide to think of it than on how it may look to us. Where perceptual judgments are employed in mathematical arguments, they are almost invariably assessments of model images or forms meant explicitly to stand in for an abstract infinity of ideal objects: we make judgments about ideal abstract triangles based on the images we can draw on a piece of paper or a blackboard. Simple, manageable examples take a particular prominence because of their greater workability and presence in our empirical experience (Bloor 1976, 90).

Indeed, the premise in meaning-finitist epistemology that knowledge is empirical and inductive puts a heavy premium on the immediate, familiar, and workable. In mathematics, objects are said to exist on a full scale of conceptual accessibility from the most basic ostensible representations, to less basic but still operable and workable formalisms and examples, all the way to the infinities of utterly ungraspable ideal objects. Strong program treatments of mathematics focus on precisely the simplest and most immediate cases because, in addition to being more accessible for sociologists and philosophers, they are presumed from the start to be at the heart of the greater problematics of mathematical knowledge. If the simplest and most

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painted here, Bloor nonetheless makes sure to emphasize that mathematical knowledge depends inescapably on the interpretation of *both* natural and social kinds, and hence both P/N and S/S devices.

obvious concepts are fundamentally social in their constitution, the reasoning goes, then so must be the more complex ones.

As Lakatos illustrates, the social negotiations at play in our knowledge of 2+2 reappear at nearly every juncture in the attempt to characterize polyhedra. If 2+2 is irreducibly social (and meaning finitists would point to the considerable room for variation in the meaning and use of numbers and the operations of arithmetic to insist that it is), then the compounded conventional character of polyhedra should be doubly ineliminable. For 2+2 is a simple formalism tied to simple ostensive experience, whereas "polyhedron" is a highly mediated concept encompassing a range of heuristics, formalisms, and heuristics and formalisms about those heuristics and formalisms. Because the "polyhedron" concept comes from a much broader tension, it must fit each instance that much more slackly, and be that much more elastic. At the same time, meaning finitists risk a certain amount of question-begging. Put crudely, it is not surprising that an approach beginning with the central importance of simple examples should conclude that those same examples are centrally important in the constitution of (mathematical) knowledge. From a meaning-finitist perspective, this does not present a problem, for it is taken as a matter of principle that all concepts have the same sorts of epistemic foundations and are formally indistinguishable.

But in a more conventional view, there is a substantial difference between the knowledge of 2+2 and that of V-E+F. The former is a formalism understood to stand in for an infinity of possible empirical instantiations. The latter is also a formalism, but at its root it is a formalism about formalisms. It describes a putative relation among an infinity of polyhedra, themselves comprehended as mixtures of empirical images and formalisms. Moreover, all but a few of the ideal polyhedra to which Euler's theorem is taken to apply are known from the start to be utterly unrealizable. At the root of the problem of induction is the question of what can count as future instances of a phenomenon.

Platonism

As mathematical claims and concepts grow more complex, their significance emanates more and more from mathematicians' ability to reason with and extend them beyond their initial contexts of enunciation. Euler's theorem is not just about images of cubes, though such images are prominent at the beginning of Lakatos's narrative. Rather, Euler's theorem is about a vast constellation of polyhedra, both realized and unrealized. Just as it is possible for a new theorem to contradict "already known" examples, a mathematician may also use a theorem to make statements about "as yet unknown" examples with some justifiable confidence. It is even possible for theorems to imply meaningful assertions about objects which are not strictly knowable, such as polyhedra too complex to characterize explicitly. Lakatos shows the problems for mathematical objects at their conceptual boundaries, but it can be argued that every single formalism in his account nonetheless applies completely unproblematically to an infinite collection of ideal polyhedra in what might be called the concept's conceptual interior. Hospitals stock the range of pyjamas that they do because those pyjamas will fit most patients without difficulty.

This view is a form of mathematical Platonism, a matter of central concern in SSK (e.g. Bloor 1983, 83). Platonism is both an ontological and an epistemological position. Ontologically, it holds that mathematical objects exist independent from human interventions. There will always be ideal pyramids, for example, and it will always be the case that V-E+F=2 for them, regardless of where the Ancient Egyptians buried their royalty or whether Euler performed any calculations or made any conjectures. Epistemologically, it holds that we can obtain an (albeit always imperfect) knowledge of such ideal mathematical objects through reasoned deductions. In both respects, Platonism overlaps substantially with realism, another of SSK's pivotal touchpoints, which holds that objects in nature exist independent of our experience of them (e.g. Bloor 1973, 176; BBH 1996, 88, *et passim*).

Unsurprisingly, meaning finitists are fundamentally skeptical of Platonism. Orthodox Platonism implies that the collection of objects to which a mathematical rule or statement applies is fixed in advance and that concepts have stable essences, contradicting the flexibility tenets of finitism (Barnes 1982, 32; Bloor 1983, 28-29, 88; BBH 1996, 85; Bloor 1997, 37, 130). With its implication that some statements are pre-given as correct, Platonism undermines SSK's principle of symmetry—that agreement about theories should be explained with the same social mechanisms, whether those theories are deemed true or false (Bloor 1973, 176-177; Bloor 1997, 36).

But the meaning-finitist objection to Platonism comes with a twist. While there is no hope of reconciling meaning-finitism and Platonism on epistemological grounds, the distinction between Platonist ontology and Platonist epistemology allows scholars of SSK to dispense with the latter without altogether doing away with the former. Platonist epistemology, they argue, is irredeemably circular. Even if there are essential truths about ideal objects in mathematics, one cannot know that one has found them except through social mechanisms like argumentation and demonstration (Bloor 1973, 182; Bloor 1983, 86). Meaning finitists uniformly reject Platonist epistemology, just as they reject its realist cousin.

When it comes to ontology, however, Platonism becomes just one of many possible outlooks, implying its own special set of strategies for managing knowledge (Bloor 1997, 38). The sociologist of science need not judge the ultimate reality of the objects of knowledge, but should rather study the social function of positing certain things as real (Bloor 1973, 190; Barnes 1982, 82). We behave differently toward pyjama-wearing entities in the hospital because we believe them to be humans with life histories and everything else that entails, and (as long as we believe that) it would not much change our behavior if those entities were *really* mechanical automata or particularly convincing sacks of rags underneath. That is not to deny their humanity, but to say that when it comes to our actions it is our socially-defined belief in their humanity that really counts, rather than any purported underlying reality of humanity. In the case of polyhedra, it is not necessary to say whether or not there are such things as ideal polyhedra or eternally valid theorems about them that are independent of human activity. Rather, one starts from the perspective that mathematicians are human, and that as humans they make a variety of claims (Platonist and realist ones) that shape the mathematical objects and theorems under discussion. One does not say whether or not ontological Platonism is valid while at the same time recognizing that such Platonism is a widely held view that fundamentally shapes the kinds of knowledge claims mathematicians can and do make.

Indeed, meaning finitists seem perfectly happy to endorse a "naive common-sense realism" positing an independent reality but, in keeping with the ontology-epistemology distinction, "refusing to conflate external reality with anything that is said of it" (BBH 1996, 88). For Barnes (1982, 79), realism with respect to nature is directly analogous to Platonism in mathematics, and the latter is even used as a justification of the former. On evaluating a large number of otherwise unproblematic polyhedra and finding that for several of the more complicated ones V-E+F came out to three, one would suppose oneself to have miscounted or to have inadequately represented the object rather than reject the original proposition. As Barnes suggests, just because some eggs would fall out differently on different runs through an egg sorting machine it is not necessary to reject the proposition that eggs come in different sizes (Barnes 1983, 541). Strong relativist claims about *how* we know remain, in this sense, impartial with respect to *what* we know.

In this sense, it is still consistent with meaning finitism to claim, as is famously attributed to mathematician Henri Poincaré, that geometry is "the art of reasoning correctly about figures which are poorly constructed" (see Netz 1999, 33-34; Hardy 1967 [1940], 125). One has only to admit the instantiating role of figures without claiming for them unmediated access to any essence in what they depict. In terms of their process of elaboration, mathematical objects always go beyond their Barnesian tension of instances, not by having an ultimate (ontological) essence but by being (epistemically) flexible, yet relational, in their future applications. A putative ideal shape with millions of vertices, edges, and faces can, as a practical matter, only count as a Platonic polyhedron to the extent that it is possible to compare it with the simple rules, figures, and heuristics in the polyhedron-concept's tension.

That is, there is no immediate way to verify one way or another whether such a shape *really* exists, but meaning finitism helps us account for the epistemic process by which mathematicians can confidently believe that it does, as well as for the effects that belief has on their claims and practices. Polyhedra, whether they exist Platonically or not, maintain a social existence compatible with Platonism by virtue of their meaning-finitist constitution. Because mathematicians understand polyhedra through piecemeal experiences, intuitions, models, and heuristics, it is always possible to suppose that there is some unified ideal concept underneath it all. Polyhedra are pyjamas all the way down, but that is precisely what makes it possible to believe in the reality of that which the pyjamas classify. As with our account of induction in meaning-finitist mathematics, we again find simple workable examples at the core of concept-formation.

Conclusion

In the end, Barnes's pyjama analogy reminds us that labels must be made to fit their objects. Hospital pyjamas are loose-fitting but not omni-fitting. Similarly, in Lakatos's story the "polyhedron" label has an inherent flexibility that allows genuine differences of understanding over what can count as a polyhedron without implying that any object whatsoever could readily be so-labeled. Indeed, the inductive character of meaning-finitist mathematical knowledge suggests that the negotiations that must occur at the contested boundaries of the polyhedron-concept can only apply to simple, workable candidate polyhedra. We can understand V-E+F to equal two for infinities of unpicturable polyhedra, but in the cases where the polyhedron concept really counts—those cases at the heart of mathematical research and understanding—the exemplars in question must be representable in ways that make the label accessible and meaningful.

In an important practical sense, most ideal polyhedra on the interior of the polyhedron-concept cannot wear pyjamas—there is no lived situation in which they can be directly manifested, manipulated, addressed, and labeled. Such wholly-putative polyhedra subsist unproblematically precisely insofar as they are never called into question. Indeed, most of them cannot, as a matter of practice, be put to the test. The meaning-finitist account of Platonism helps us recognize that most polyhedra exist in a meaningful sense only because they can be posited in relation to workable guiding instances. These latter instances correspond to what Lakatos seems to indicate with the term "heuristics." We tend to think of heuristics as guideposts to understanding what mathematics is really about, but for a meaning finitist all we can really know and explain are the guideposts themselves—those objects that can be issued pyjamas at all.

Nor can hospital pyjamas be made to stretch and pull without bearing traces of such contortions. Mathematicians generate new concepts and refine old ones by balancing the familiar with the unfamiliar, and each decision about the scope and meaning of an idea affects what is possible for future ones. Like pyjamas, labels can tear at the seams. Old mathematical frameworks or definitions can prove untenable in the face of contradictory intuitions or heuristics. Concepts can fail to win approval under the weight of countervailing arguments—this is Lakatos's process of monster barring. In each case, classical SSK meaning finitism affords an enriched view of the resources and processes that make conceptual adaptation in mathematics possible.

As an alternative to epistemic Platonism, SSK meaning finitism forces renewed attention to the role simple examples play in even the most complex mathematical knowledge. This focus on simple objects and their labels, born as much out of methodological necessity as principled conviction, yields under further consideration a robust framework for interrogating the mathematical uses and meanings of models and heuristics. It helps one take account of what is manifestly evident in mathematics both past and present: that the discipline's objects and notions are shaped, challenged, and manifested through the social interactions of mathematicians.

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Sources

Michael J. Barany (2014) "Savage Numbers and the Evolution of Civilization in Victorian Prehistory," *British Journal for the History of Science* 47(2): 239-255.

Barry Barnes (1974) *Scientific knowledge and sociological theory.* London: Routledge.

— (1981) "On the Conventional Character of Knowledge and Cognition," *Philosophy of the Social Sciences* 11(3): 303-333.

— (1982) *T. S. Kuhn and Social Science.* London: Macmillan.

— (1983) "Social Life as Bootstrapped Induction," *Sociology* 17(4): 524-545.

Barry Barnes, David Bloor, and John Henry (1996) *Scientific Knowledge: A Sociological Analysis.* London: Athlone.

David Bloor (1973) “Wittgenstein and Mannheim on the Sociology of Mathematics,” *Studies in History and Philosophy of Science* 4(2): 173-191.

— (1976) *Knowledge and Social Imagery.* London: Routledge.

— (1978) “Polyhedra and the Abominations of Leviticus,” *The British Journal for the History of Science* 11(3): 245-272.

— (1983) *Wittgenstein: A social theory of knowledge*. London: Macmillan.

— (1992) “Left and Right Wittgensteinians,” in *Science as Practice and Culture*, ed. A. Pickering. Chicago: University of Chicago Press, 266-282.

— (1997) *Wittgenstein, Rules and Institutions*. London: Routledge.

Ian Hacking (2014) *Why is there Philosophy of Mathematics at all?* Cambridge: Cambridge University Press.

G. H. Hardy (1967 [1940]) *A Mathematician's Apology.* Cambridge: Cambridge University Press.

Imre Lakatos (1976) *Proofs and Refutations: The Logic of Mathematical Discovery,* ed. John Worrall and Elie Zahar. Cambridge: Cambridge University Press.

Michael Lynch (1992a) “Extending Wittgenstein: The Pivotal Move from Epistemology to the Sociology of Science,” in *Science as Practice and Culture*, ed. A. Pickering. Chicago: University of Chicago Press, 215-265.

— (1992b) “From the ‘Will to Theory’ to the Discursive Collage: A Reply to Bloor's ‘Left and Right Wittgensteinians’,” in *Science as Practice and Culture*, ed. A. Pickering. Chicago: University of Chicago Press, 283-300.

Reviel Netz (1999) *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History.* Cambridge: Cambridge University Press.